COMPUTER GRAPHICS AND IMAGE PROCESSING 8, 369–381 (1978)

Texture Synthesis Using a Growth Model

RYUZO YOKOYAMA

Iwate University, Japan

AND

ROBERT M. HARALICK

Department of Computer Science and Electrical Engineering, University of Kansas, Lawrence, Kansas 66045

Received December 9, 1977

A systematic method of gray tone texture generation is presented. The procedure is composed of two phases. The first phase is to generate a set of images having characteristic pattern structures. Called mother images, these are synthesized by repetitive applications of seed distribution operations, skeleton growth operations, and muscle growth operations. The second phase is to synthesize a set of gray tone textures from a mother image by applying specific probabilistic transformations which convert numbers in the mother image to gray tone values.

1. INTRODUCTION

Texture pattern feature extraction by computer is a very important technique in the analysis of remotely sensed data. Many algorithms have been suggested for the purpose. Comparative studies of the efficiency and accuracy of these techniques have been done to identify their advantages [1, 2]. In most such studies, samples of textures are analyzed from relevant natural texture patterns of real image data. However, sometimes these results might not be generalizable due to the limited number and variety of the samples.

In order to proceed with comparative studies in a more thorough manner, we can supplement the real data by synthesizing texture patterns having specified statistics. There are only a few reports about such texture pattern synthesis processes [3, 4, 5]. Recently Schachter [6] *et al.* suggested a method based on random mosaic models.

In a former paper [7], the authors suggested a synthesis method of texture patterns using regular Markov chains. The spatial cooccurrence probabilities of synthesized textures can be described by the transition matrices of the Markov chains. The texture patterns generated by this method have a spatial gray tone microstructure.

0146-664X/78/0083-0369$02.00/0

This paper presents another systematic method of texture pattern generation. This method is originally aimed at synthesizing a set of textures having spatial gray tone macrostructures in their patterns. In the first stage of the procedure, a symbolic image is synthesized by applying various operations called seed distribution operations, skeleton growth operations, and muscle growth operations. Then a numeric image is generated by applying a transformation from the category symbols of the symbolic image into gray tone values. Synthesized patterns depend on the combination of operations and their parameters during the symbolic image generation and the final transformation.

The second phase is to synthesize a set of numeric images whose texture patterns and spatial cooccurrence probability matrices [8] of gray tones are similar to those of the image generated in the first phase. This is done by applying specific probability transformations to gray tone values of the original image. For convenience, the original and its derived images are called a mother and relative images, respectively. The methods of generating a mother image and a relative image are described in Sections 2 and 3, respectively.

2. MOTHER IMAGE GENERATION

Before the main discussion, it will be appropriate to explain the notation used in this paper. The domain of the image is assumed to be rectangular. Let $L_r = \{1, 2, \ldots, N_r\}$ and $L_c = \{1, 2, \ldots, N_c\}$ be the row and the column spatial domains, respectively. Then $L_r \times L_c$ is the set of resolution cells of digital images. A digital image can be defined as a function

$$I: L_r \times L_c \rightarrow G$$

where $G = \{1, 2, \ldots, n\}$ is assumed to be a set of gray tone values or a set of category symbols. In the former case, the image is called a numeric image. In the latter case, it is called a symbolic image.

The mother image generation simulates growing processes in nature, for example plants, biological organs, geological terrains, etc. It is composed of seed distribution, skeleton growth, and muscle growth. Then a gray tone is generated by transforming the category symbols in the symbolic image into gray tone values.

2.1. Seed Distribution Operation

The image domain is assumed to be white at the beginning so that all resolution cells have the same category symbol, say 1. The first operation for symbolic image generation is seed distribution. This changes the category symbols of some resolution cells in the image domain. The ones whose category symbols are changed are called seed cells. Of course, it is possible to have many varieties of seeds and a different category symbol is associated with each variety.

Allocation of seed cells can be performed either in a deterministic manner or in a random manner. In the deterministic manner, seeds are arranged periodically in the spatial domain. Only the position of the seed cell having the

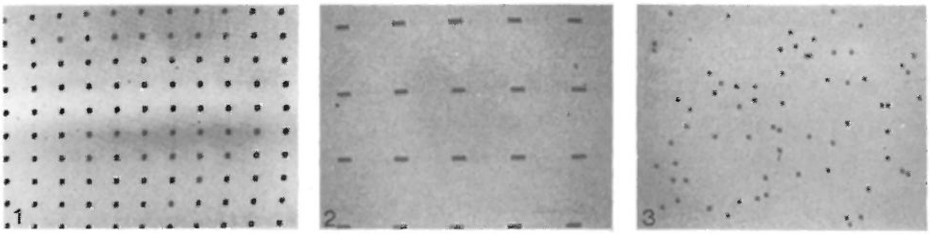

FIG. 1. Samples of seed image. Seed 1: Seeds exist at coordinates of $(10i - 5, 10j - 5)$ with $i, j = 1, 2, \ldots$. Seed 2: Seeds exist at coordinates of $(20i - 10, 20i - 9)$, $(20i - 10, 20i - 10)$, $(20i - 10, 20i - 11)$ and $(20i - 10, 20i - 12)$ with $i, j = 1, 2, \ldots$. Seed 3: The amount of distributed seeds is 1% of the total resolution cells in the image.

smallest spatial coordinate and its periodicity in the row and the column directions need be specified.

In random seed distribution, positions of seeds are determined by pseudorandom numbers with respect to a probability function of seed location in the image domain. The probability functions of seed location and total number of distributing seeds must be specified for each species.

Samples of seed images are shown in Fig. 1. Images of seeds 1, 2, are generated by the deterministic mode, and of seed 3 by the random mode. In each case, there is only one species of seeds (black dots).

2.2. Skeleton Growth Operation

The purpose of the skeleton growth operation is to extend skeleton shapes from seeds. As in the case of the seed distribution operation, there is a deterministic mode and a random mode in this operation.

In the deterministic mode, skeletons with definite shapes grow from each seed cell. The shapes of skeletons can be arbitrarily specified with respect to the seed species. Then skeletons extend their shapes one resolution cell at a time according to the specified sequences during the scanning iteration of the execute cycle. Resolution cells forming skeleton shapes are called grown cells. Category symbols of grown cells can be specified arbitrarily.

In the random growth mode, skeletons extend their shapes randomly from bud cells during each scanning iteration. Bud cells are resolution cells from which skeletons can extend their shapes. In the first scanning iteration, bud cells are directly assumed to be all seed cells, or can be limited to seed cells of specific species. When a bud cell of a skeleton is located in a scanning iteration, a random number generator is accessed. Its output specifies one of eight directions (0, 45, ..., 315) of growth, or growth halting.

If it specifies a specific growth direction, the skeleton extends its shape to the next resolution cell in that direction. The bud cell is renamed to be a regular grown cell, and the new grown cell is appointed a bud cell for the next scanning iteration. There are three cases when growth is halted. One is the case when the random number specifies growth halting. The other two cases are when an extended part hits a grown cell on the edges of the image domain.

In these cases the bud cells are simply renamed to be regular grown cells, and the skeleton cannot grow from them at the next scanning iteration.

Different probability functions of growth direction can be associated with different seed species. Category symbols can be arbitrarily assigned to skeletons according to their species. Although skeletons grown by this procedure have arbitrary shapes, they can be classified into groups by the category symbols of grown cells or by their shape forms. The execution of the growth operation stops when a preassigned number of scanning iterations are completed.

Figure 2 shows samples of skeleton images generated from seed distributions 1 and 3 of Fig. 1 by the random mode. In each case, the number of scanning iterations is 5.

Figure 3 shows samples of images generated by the deterministic mode of the skeleton growth operations. In their seed images, two species of seeds are distributed. Then skeletons with two gray tone values (black, gray) are extended from these seeds.

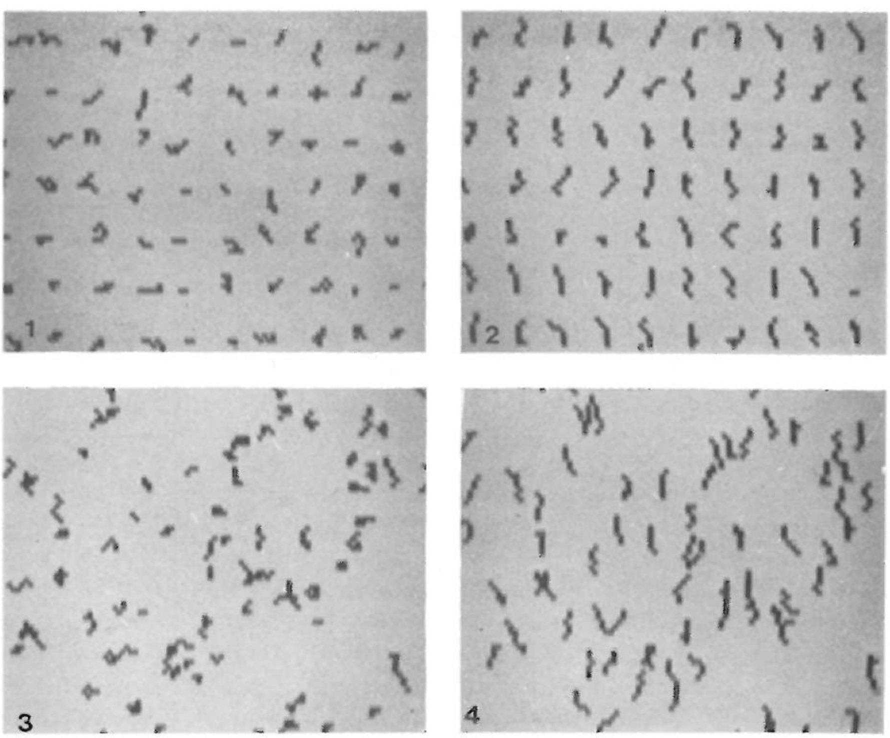

FIG. 2. Samples of skeleton images generated by the random mode. For skeletons 1 and 3, the probability vector of growth directions is $P_1 = [0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.0]$. For skeletons 2 and 4, the probability vector of growth directions is $P_2 = [0.05, 0.0, 0.0, 0.0, 0.05, 0.2, 0.5, 0.2, 0.0]$. Skeleton 1: Generated from seed 1. Skeleton 2: Generated from seed 1. Skeleton 3: Generated from seed 3. Skeleton 4: Generated from seed 3.

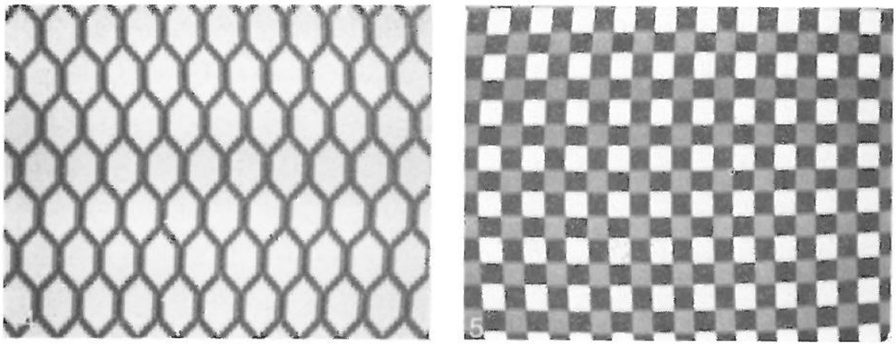

FIG. 3. Samples of skeleton images generated by the deterministic mode. Skeleton 4: Honeycomb image. Skeleton 5: Check image.

2.3. *Muscle Growth Operation*

The muscle growth operation is very similar to the random skeleton growth operation. The only difference is that every grown cell is identified as a bud cell in each scanning iteration of growth, whereas bud cells in the skeleton growth are limited among grown cells. The grown parts can be likened to muscles rather than skeletons. Applied to an image of skeletons, the speed of shape growth in this operation is very rapid.

Different probability functions of growth directions can be specified with respect to the seed species of skeletons or the scanning iterations to provide various features of muscle shapes. The category symbols of muscles can be arbitrarily assignable so that layers of muscles can be formed. When a preassigned number of scanning iterations is completed, growth stops.

Figure 4 shows samples generated from the skeleton images in Fig. 3 by the muscle growth operation. Eight category symbols, from the blackest to the lightest, are used in each image. By directly associating the category symbols to gray tone values, we can see that combinations of different parameters and operations can provide a variety of texture patterns.

2.4. *Numeric Image Generation Operation*

The final stage of the texture synthesis generation is the generation of a numeric image from the symbolic image which is produced by the operations described in the previous sections. This is done by transforming the category symbols of the symbolic image into gray tone values.

The transformation can be performed either in a deterministic manner or in a random manner. In the former, a table making the correspondence between category symbols and gray tone values must be set up. In the latter case, conditional probabilities of gray tone values given each category symbol must be specified.

Figure 5 shows samples of mother images of texture patterns generated from Texture 9 in Fig. 4 by the deterministic mode. R_1, R_2, and R_3 are the applied transformation rules from the category symbols to gray tone values. These

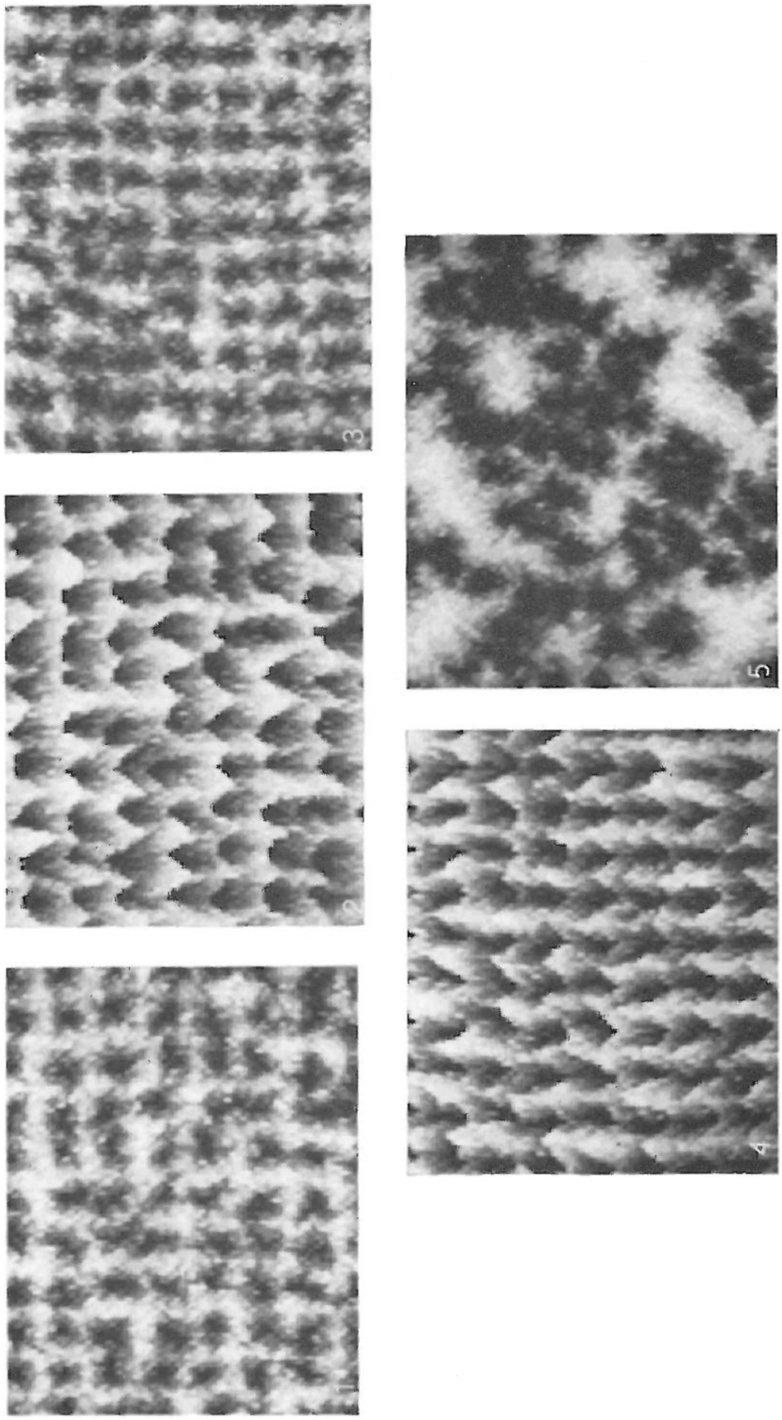

FIG. 4. Samples of texture patterns synthesized by the muscle operation, where $P_1 = [0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.0]$ and $P_2 = [0.05, 0.0, 0.0, 0.0, 0.05, 0.2, 0.5, 0.2, 0.0]$. Texture 1: Generated from Skeleton 1 with P_1. Texture 2: Generated from Skeleton 1 with P_2. Texture 3: Generated from Skeleton 2 with P_1. Texture 4: Generated from Skeleton 2 with P_2. Texture 5: Generated from Skeleton 3 with P_1.

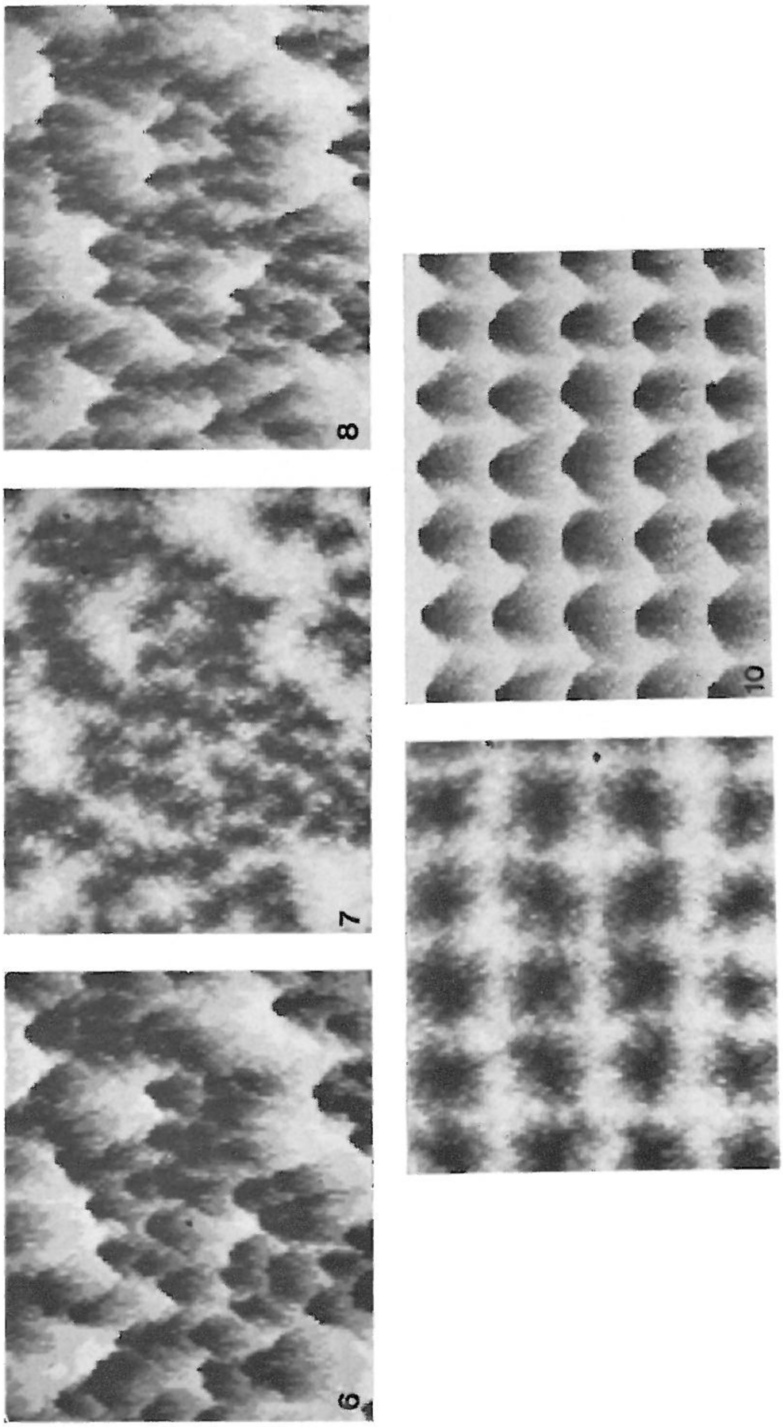

FIG. 4. (*continued*). Texture 6: Generated from Skeleton 3 with P_2. Texture 7: Generated from Skeleton 4 with P_1. Texture 8: Generated from Skeleton 4 with P_2. Texture 9: Generated from Seed 2 with P_1. Texture 10: Generated from Seed 2 with P_2.

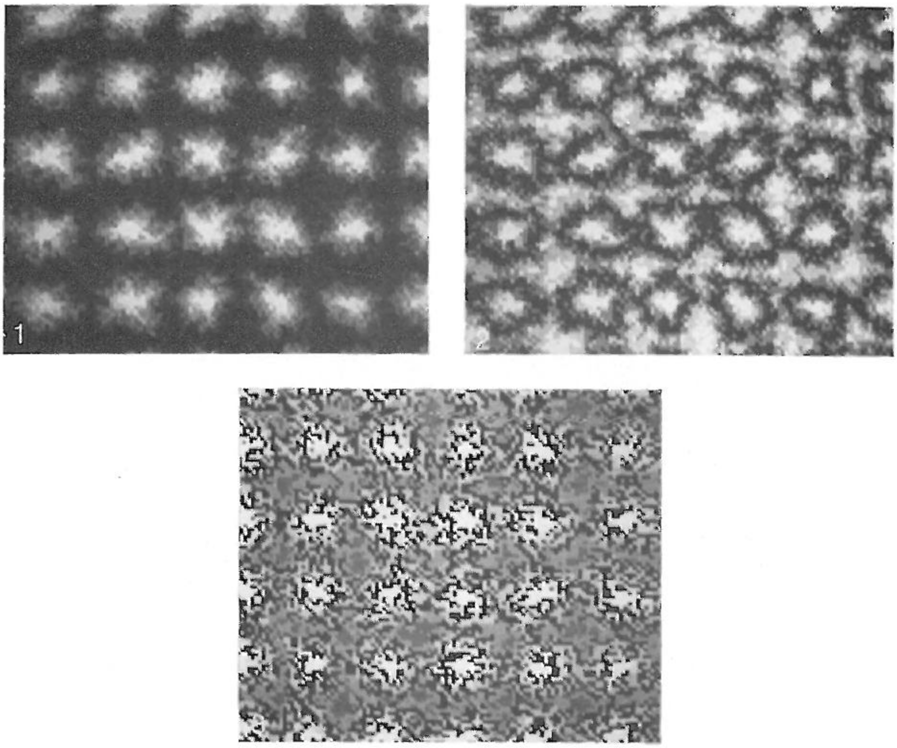

FIG. 5. Samples of mother images generated from Texture 9 by various transformation rules. Mother 1: Generated by $R_1 = [8, 7, 6, 5, 4, 3, 2, 1]$. Mother 2: Generated by $R_2 = [4, 3, 2, 1, 1, 2, 3, 4]$. Mother 3: Generated by $R_3 = [8, 1, 7, 2, 6, 3, 5, 4]$.

rules are described as vectors

$$R_1 = [8, 7, 6, 5, 4, 3, 2, 1],$$
$$R_2 = [4, 3, 2, 1, 1, 2, 3, 4],$$
$$R_3 = [8, 1, 7, 2, 6, 3, 5, 4].$$

where each integer in the ith component, $i = 1, 2, \ldots, 8$ indicates the gray tone value to correspond with category symbol i. It may be noted that the mother image generated by the transformation rule

$$R = [1, 2, 3, \ldots, 8]$$

is identical to Texture 9 itself.

Until now we have discussed the generation process of a mother image. Although the first step and the last step of the generation process must be consistent with a seed distribution operation and a numeric image generation operation respectively, operations of seed distribution, skeleton growth, and muscle growth are applicable repetitively during the intermediate stages. The texture pattern of a generated image depends on the sequence of operations and their specified parameters. The variety of generated pattern textures can be expected to be quite large. In practice, a computer program to generate a mother image can be simply constructed by combining these subroutines in a molecular structure.

3. RELATIVE IMAGE GENERATION

A relative image is defined as a numeric image whose spatial cooccurrence probability matrix is derived from that of the mother image by applying a probabilistic transformation to the mother image. Let us consider the statistics of an image generated by a probability transformation from a mother image. Assume that the spatial cooccurrence probability matrix of the mother image is P and the probabilistic transformation is specified by the matrix

$$Q = \begin{bmatrix} q(1, 1) & \dots & q(1, 2) & \dots & q(1, n) \\ q(2, 1) & \dots & q(2, 2) & \dots & q(2, n) \\ \vdots & & \vdots & & \vdots \\ q(n, 1) & \dots & q(n, 2) & \dots & q(n, n) \end{bmatrix} \tag{1}$$

where each $q(i, j)$ is the conditional probability of generating value j at a resolution cell in the transformed image when the resolution cell of the mother image has value i. The matrix Q must satisfy

$$\text{(i)} \qquad q(i, j) \geq 0, \qquad \text{for} \qquad i, j = 1, 2, \dots, n$$

and

$$\text{(ii)} \qquad \sum_{j=1}^{n} q(i, j) = 1 \qquad \text{for} \qquad i = 1, 2, \dots, n.$$

Let $P(i, j)$, each element of P, be the probability that values i and j occur next to each other on the mother image, and let $r(k, h)$ be the probability that values k and h occur next to each other on the relative image. Then

$$\begin{aligned} r(k, h) &= \sum_{i=h}^{n} q(i, k) \{ \sum_{j=1}^{n} p(i, j) q(j, h) \} \\ &= Q_k{}^{\mathrm{T}} P Q_h. \end{aligned} \tag{2}$$

where Q_k, $k = 1, 2, \dots, n$ is the kth column of Q and the superscript T means the transpose of a matrix. The spatial cooccurrence probability matrix R of the transformed image has $r(k, h)$ as its entry and is defined as

$$R = Q^{\mathrm{T}} P Q. \tag{3}$$

Thus, the spatial cooccurrence probability matrices of the mother image and the relative image define a congruence relation associated with the probability matrix Q.

The occurrence probability $r(k)$ of a gray tone value k in the transformed image is the sum of entries on the kth row of R; the occurrence probability vector $\hat{R}$ of gray tone values is given by

$$\hat{R} = \begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ r(n) \end{bmatrix} = Q^{\mathrm{T}}PQ \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = Q^{\mathrm{T}}P \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

and finally

$$\hat{R} = Q^{\mathrm{T}}\hat{P} \tag{4}$$

where $\hat{P}$ is the occurrence probability for the mother image. Thus the probability vector of the transformed image is provided by applying a linear transformation Q^{T} to the occurrence probability vector of the mother image.

For generation of a relative image, the matrix Q for the transformation is conveniently described to be

$$Q = I + \Delta Q \tag{5}$$

where I is the unit matrix and ΔQ is

$$\Delta Q = \begin{bmatrix} -\Delta q_{11} & \Delta q_{12} & \cdots & \Delta q_{1n} \\ \Delta q_{21} & -\Delta q_{22} & \cdots & \Delta q_{2n} \\ \vdots & & & \vdots \\ \Delta q_{n1} & \Delta q_{n2} & \cdots & -\Delta q_{nn} \end{bmatrix} \tag{6}$$

and satisfies

(i) $$\sum_{i \neq j} q_{ij} = \Delta q_{ii} \leq 1$$

and

(ii) $$\Delta q_{ij} \geq 0.$$

A relative image is defined to be a transformed image from the mother image associated the matrix Q such that $\Delta q_{ii} = i = 1, 2, \ldots, n$ are constrained to be small quantities. From (3) and (4), the statistics of the relative image are

$$R = (I + \Delta Q)^{\mathrm{T}}P(I + \Delta Q) = P + \Delta P \tag{7}$$

where

$$\Delta P = \Delta Q^{\mathrm{T}}P + P\Delta Q + \Delta Q^{\mathrm{T}}P\Delta Q \tag{8}$$

and

$$\hat{R} = (I + \Delta Q)^{\mathrm{T}}\hat{P} = \hat{P} + \Delta\hat{P} \tag{9}$$

where

$$\Delta\hat{P} = \Delta Q^{\mathrm{T}}P.$$

ΔP and $\Delta\hat{P}$ are the amounts of the perturbation to the spatial cooccurrence probability matrix and the occurrence probability vector, respectively. When the Δq_{ii} are kept small, ΔP and $\Delta\hat{P}$ are constrained to small values also.

A set of relative images can be generated by probability transformations using various ΔQ matrices which are admissible as small perturbations. The visual distortions of texture patterns gradually increase as Δq_{ii}, the (i, j)th entry of ΔQ, increases. By restricting Δq_{ii} to small values we can get a set of texture patterns which are both shape and statistically oriented to the mother image.

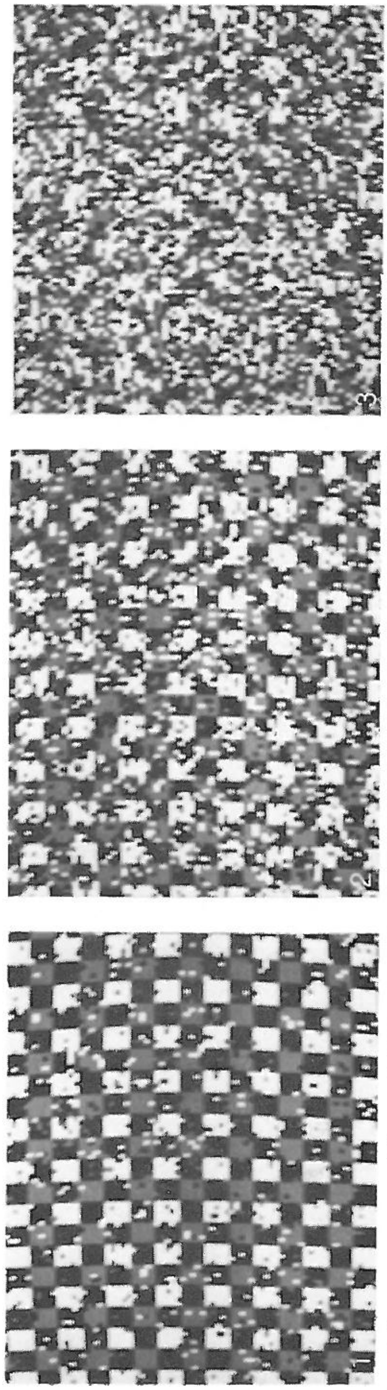

Fig. 6. Images generated by probability transformation to Skeleton 4. RS-1; $\Delta q_{ii} = 0.1$ and $\Delta q_{ij} = 0.05$ for $i \neq j$. RS-2; $\Delta q_{ii} = 0.25$ and $\Delta q_{ij} = 0.125$ for $i \neq j$. RS-3; $\Delta q_{ii} = 0.5$ and $\Delta q_{ij} = 0.25$ for $i \neq j$.

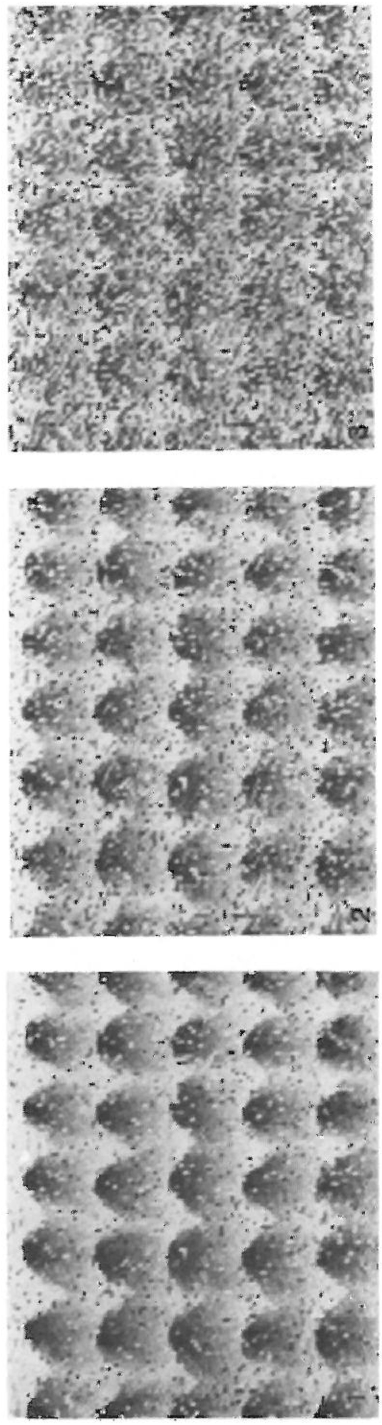

FIG. 7. Images generated by probability transformation to Texture 10. RM-1; $\Delta q_{ii} = 0.14$ and $\Delta q_{ij} = 0.02$ for $i \neq j$. RM-2; $\Delta q_{ii} = 0.28$ and $\Delta q_{ij} = 0.04$ for $i \neq j$. RM-3; $\Delta q_{ii} = 0.56$ and $\Delta q_{ij} = 0.08$ for $i \neq j$.

Figures 6 and 7 are texture images generated from Muscle 11 and Skeleton 5, respectively, by applying probability transformations. The values of ΔQ for the transformations are given in the captions. As the diagonal elements in ΔQ for transformation become relatively large, the distortion of the texture patterns from each original image increases.

4. CONCLUSION

A systematic method of generating gray tone texture patterns has been described. The first step of the procedure is to synthesize a mother image by a repetitive application of specific operations called seed distribution, skeleton growth, muscle growth, and probability transformation. Combinations of those operations and their parameters give the texture patterns of synthesized images a large variety. The second step is to synthesize a set of textures whose patterns and spatial cooccurrence matrices are related to those of the mother image. These are called relative images and are generated by applying specific probability transformations to the values of the mother image.

REFERENCES

1. L. Kirvida and G. Johnson, "Automatic Interpretation of Earth Resources Technology Satellite Data for Forest Management." Symposium on Significant Results Obtained from the Earth Resources Technology Satellite, NASA SP-327, March 1973, pp. 1076–1082.
2. J. Weszka, C. Dyer, and A. Rosenfeld, "A Comparative Study of Texture Measures for Terrain Classification," *IEEE Trans. Syst., Man, Cybernetics*, **SMC-6**, No. 4, 1976.
3. A. Rosenfeld and B. Lipkin, "Texture Synthesis, Picture Processing and Psychopictories" (B. Lipkin and A. Rosenfeld, Eds.), Academic Press, New York, 1970.
4. Julesz, "Visual Pattern Discrimination," *IEEE Trans. Inform. Theory*, 1962, 84–92.
5. McCormick *et al.*, "Time Series Model for Texture Synthesis," *Internat. J. Comput. Inform. Sci.*, **3**, 1974, 329–343.
6. B. Schachter, A. Rosenfeld, and L. Davis, "Random Mosaic Models for Texture," Computer Science Technical Report, TR-463, July 1976, University of Maryland.
7. R. Yokoyama and R. Haralick, "Texture Pattern Image Generation by Regular Markov Chain," in preparation.
8. R. Haralick, K. Shanmugam, and I. Dinstein, "Textural Features for Image Classification," *IEEE Trans. Syst., Man, Cybernetics*, **SMC-3**, 1973, 610–621.
9. J. Kemeny and J. Snell, "Finite Markov Chains," Springer–Verlag, New York, 1960.