Machine Vision and Applications (1989) 2:1–16

Machine Vision and Applications

Performance Assessment of Near-Perfect Machines

Robert M. Haralick
Intelligent Systems Laboratory, University of Washington, Seattle, Washington, USA

Abstract: This is a short, practical note which provides some reference operating curves of false acceptance rate versus missed acceptance rate as a function of N, the number of test samples and f_0, the specified machine error rate requirement. The only important statistical assumption made is the statistical independence of the samples in the test. The analysis shows that, to equalize the false acceptance rate with the missed acceptance rate, the machine acceptance test must use a threshold $K^* = Nf_0 - 1$. If there are K^* or fewer failures, then the machine acceptance test is passed. Otherwise, it fails. Furthermore, with such an acceptance test, the probability that the test is accurate depends only on the product Nf_0. When $Nf_0 = 10$, the probability that the test is accurate is 0.875. When $Nf_0 = 20$, the probability that the test is accurate is 0.912. These results indicate the necessity of large sample sizes when performing acceptance testing of near-perfect machines whose required error rate f_0 is very close to zero.

Key Words: performance assessment, error rate, acceptance rate, rejection rate, performance testing

1 Introduction

Machines which are employed in recognition or defect inspection tasks are required to perform nearly flawlessly. Being sure that a machine meets performance specifications can require assessing its performance on a much larger sample than intuition might lead one to believe. And depending on what is meant by "meeting specifications" being sure that a machine meets specification can require that its performance, which is sampled in a limited assessment test, be better than intuition might lead one to believe. This note explains why.

Address reprint requests to: Robert M. Haralick, Intelligent Systems Laboratory, Dept. of Electrical Engineering, FT-10, Univ. of Washington, Seattle, WA 98195, USA.

To set the stage for the explanation, consider the action of a recognition or inspection machine. It observes a part or document which it can either accept or reject. Rejection means that the machine is unable to carry out the recognition or inspection. Judgment is reserved. Acceptance means that the machine will make a judgment. If the machine is a recognition machine, the judgment can either be a correct judgment or an incorrect judgment. The fraction of time that the machine's judgment is incorrect is called the *error rate*.

The error rate is composed of two kinds of errors: false detection errors and misdetection errors. If the machine is an inspection machine, the observed part may be flawed or unflawed and the machine's judgment may pronounce the part to be flawed or unflawed. The fraction of time that a part that is actually unflawed is judged to be flawed is called the *false detection* or *false alarm* rate. The fraction of time that a part which is actually flawed but is judged to be unflawed is called the *misdetection rate*.

In a machine which is nearly perfect, the rejection rate, the error rate, the misdetection rate, and the false alarm rate are all very small and close to zero. To determine whether a nearly perfect machine meets specification, an acceptance test must be performed. In the acceptance test, the machine is given a sample of N parts or documents to judge and the resulting number of rejects or the number of machine judgment errors or the number of misdetections, or the number of false alarms, is observed. The assessment of whether the machine meets specification then requires an appropriate comparison of the observed number from the assessment test with the performance requirement. Because the assessment tests only observes a finite sample, there is of necessity a difference between the observed performance on the test sample and the long-term performance on the total population. The issue of performance assessment then amounts to making the comparison between the specification and the observed performance in a way in which it is precisely understood what the uncertainty due to sampling is. The next section gives a derivation of the problem which then leads to a description of how this comparison can be done.

2 The Derivation

Consider the case for false alarm errors. The other cases are obviously similar. Let N, the sampling size, be the total number of parts observed and let K be the number of false alarm judgments observed to occur in the acceptance test of the N parts.

Suppose the machine performance specification indicates that its fraction of false alarms is to be less than f_0. The simplest intuitive way of making the comparison between f_0 and K is to use K in the natural manner to estimate the true false alarm rate f. The maximum likelihood estimate f of f based on K is $f = K/N$. If the estimate f of f is less than f_0, we judge that the machine passes the test. If the estimate f is greater than f_0, we judge that the machine fails the acceptance test. The issue with such a procedure is how sure are we if we apply such a procedure that the judgment we make about the machine's performance is a correct judgment. To answer this issue we must estimate the performance of our judgment. We start from the beginning.

To carry out the estimation, we suppose that, conditioned on the true error rate f, the machine's judgments are independent and identically distributed. Let X_n be a random variable taking the value 1 for a false alarm and taking a value 0 otherwise, when the machine is judging the n^{th} part. In the maximum likelihood technique of estimation, we compute the estimate f as the value of f which maximizes

$$Prob\left(\sum_{n=1}^{N} X_n = K \mid f\right) = \binom{N}{K} f^K (1-f)^{N-K}.$$

Taking the partial derivative with respect to f and setting the derivative to zero results in $f = K/N$, the natural estimate of f.

Suppose that we adopt the policy of accepting the machine if $f \leq f_0$. To understand the consequences of this policy, consider the probability that the policy results in a correct acceptance decision. The probability that $f \leq f_0$ given that $f \leq f_0$ needs to be computed.

$$Prob\,(f \leq f_0 \mid \hat{f} \leq f_0) = \int_{f=0}^{f_0} Prob\,(f \mid \hat{f} \leq f_0)\, df = \frac{\int_{f=0}^{f_0} Prob\,(\hat{f} \leq f_0 \mid f)\, Prob\,(f)\, df}{Prob\,(\hat{f} \leq f_0)}$$

To make the mathematics simple, let f_0 be constrained so that there is some integer K_0 such that $f_0 = K_0/N$. Then

$$Prob\,(f \leq f_0 \mid \hat{f} \leq f_0) = \frac{\int_{f=0}^{f_0} Prob\left(\sum_{n=1}^{N} X_n \leq K_0 \mid f\right) Prob\,(f)\, df}{\int_{f=0}^{1} Prob\left(\sum_{n=1}^{N} X_n \leq K_0 \mid f\right) Prob\,(f)\, df}.$$

The probability that the true value of f is less than or equal to f_0 given that the observed value f is less than f_0 will depend, in general, on the testor's prior probability function *Prob* (f). So, depending on the acceptance testor's prior probability function *Prob* (f), there will be some smallest number F, $0 \leq F \leq 1$, such that *Prob* $(f) = 0$ for all $f > F$. Here, the support for the prior probability function is the interval $[0, F]$.

For example, an acceptance testor who has had successful experience with previous machines from the same manufacturer might have a prior probability function whose support is the interval $[0, 2f_0]$. An acceptance testor who has had no previous experience with the manufacturer might have a prior distribution whose support is the interval $[0, 10f_0]$. An acceptance testor who has had an unsuccessful experience with a previous machine from the same manufacturer might have a prior distribution for f whose support is the interval $[0, 0.5]$.

In each of the above cases, we assume that neither we nor the testor know anything more about the prior probability function than the interval of support $[0, F]$, where we assume that $F \geq f_0$, since if not, there would be no point to perform an acceptance test to establish something we already know. In this case, we take *Prob* (f) to be that probability function defined on the interval $[0, F]$ having highest entropy. Such a *Prob* (f) is the uniform density on the interval [0, F]. Hence, we take $Prob\ (f) = 1/F,\ 0 \leq f \leq F$. Therefore

$$Prob\ (f \leq f_0 \mid \hat{f} \leq f_0) = \frac{\int_{f=0}^{f_0} \sum_{k=0}^{K_0} \binom{N}{k} f^k (1-f)^{N-k} df/F}{\int_{f=0}^{F} \sum_{k=0}^{K_0} \binom{N}{k} f^k (1-f)^{N-k} df/F}$$

$$= \frac{\sum_{k=0}^{K_0} \binom{N}{k} B(k+1, N+1-k) \cdot I_{f_0}(k+1, N+1-k)}{\sum_{k=0}^{K_0} \binom{N}{k} B(k+1, N+1-k) \cdot I_F(k+1, N+1-k)}$$

$$= \frac{\sum_{k=0}^{K_0} I_{f_0}(k+1, N+1-k) / (N+1)}{\sum_{k=0}^{K_0} I_F(k+1, N+1-k) / (N+1)}$$

$$= \frac{\sum_{k=0}^{K_0} I_{f_0}(k+1, N+1-k)}{\sum_{k=0}^{K_0} I_F(k+1, N+1-k)}$$

where $I_{f_0}(k+1, N+1-k)$ is the incomplete Beta ratio function.

The incomplete Beta ratio function can be computed from approximation such as those found in the *Handbook of Mathematical Functions* (Abramovitz and Stegun 1972). Tables for the incomplete Beta ratio can be found in Pearson (1968). Using the relation $I_f(k, N+1-k) = \sum_{x=k}^{N} \binom{n}{x} f^x (1-f)^{N-x}$, it is possible to determine the incomplete Beta ratio from tables of the cummulative binomial distribution. It is also possible to determine $I_f(k+1, N+1-k)$ from tables of the cummulative Poisson distribution function. To see how, we first note that there is a relationship between the incomplete Beta ratio function and the $\mathcal{F}$-distribution function (Johnson and Kotz 1970, p. 78)

$$I_{f_0}(k+1, N+1-k) = Prob \left(\mathcal{F}_{2(k+1),\, 2(N+1-k)} \leq \frac{f_0}{1-f_0} \frac{N+1-k}{k+1} \right).$$

For N large and f_0 small, the value of the $\mathcal{F}$-distribution can be related to a value of the χ^2 distribution.

This relationship comes about because when ν_2 is large

$$Prob\ (\mathcal{F}\nu_1\nu_2 \leq F) = Prob\ (\chi^2_{\nu_1} \leq \nu_1 F)$$

(Johnson and Kotz 1970, p. 84). Hence when N is large

$$Prob \left(\mathcal{F}_{2(k+1),\, 2(N+1-k)} \leq \frac{f_0(N+1-k)}{(1-f_0)(k+1)} \right) = Prob \left(\chi^2_{2(k+1)} \leq \frac{2(k+1) f_0 (N+1-k)}{(1-f_0)(k+1)} \right) = Prob \left(\chi^2_{2(k+1)} \leq \frac{2 f_0 (N+1-k)}{1-f_0} \right).$$

If $f_0 \ll 1$ and $k \leq N f_0$, then $2f_0(N+1-k)/(1-f_0) = 2f_0 N = 2K_0$. Therefore

$$Prob \left(\mathcal{F}_{2(k+1),\, 2(N+1-k)} \leq \frac{f_0(N+1-k)}{(1-f_0)(k+1)} \right) = Prob\ (\chi^2_{2(k+1)} \leq 2K_0).$$

Since

$$Prob\ (\chi^2_{2(k+1)} \leq 2K_0) = \sum_{i=k+1}^{\infty} e^{-K_0}(K_0)^i / i!$$

(Johnson and Kotz 1969, p. 114) we may use tables of the cumulative Poisson distribution (Pearson and Hartley 1958) and there results

$$Prob\ (f \leq f_0 \mid \hat{f} \leq f_0) = \sum_{k=0}^{K_0} \left(\sum_{i=k+1}^{\infty} e^{-K_0} K_0^i / i! \right) \Big/ \sum_{k=0}^{K_0} \left(\sum_{i=k+1}^{\infty} e^{-K_1} K_1^i / i! \right)$$

where $K_1 = FN$. When $F \gg (k+1)/(N+2)$, the value of $I_F(k+1, N+1-k) = 1$ since the variance of a Beta $(k+1, N+1-k)$ random variable will be smaller than $(k+1)/(N+1-k)^2 \ll F$ for large N. In this case, the denominator is only a few percent smaller than $K_0 + 1$. From the form of the

Poisson approximation, it is apparent that $I_f(k, N + 1 - k)$ depends only on the product fN when $N \gg 1$, $k \leq f_0 N$ and $f \ll 1$. This can also be seen directly from the formula.

Under the particular conditions we are interested in, $N \gg 100, f \ll 0.1$, and $k \ll N$. Hence $I_f(k + 1, N + 1 - k) \approx I_f(k + 1, N)$. This can be observed from the recurrence relation

$$I_x(a, b) = xI_x(a - 1, b) + (1 - x)I_x(a, b - 1).$$

Now when $a + b > 6$ and $x \ll 1$

$$I_x(a, b) \approx \phi(y)$$

where

$$y = \frac{3\{(bx)^{1/3}(1 - 1/9b) - [a(1 - x)]^{1/3}(1 - 1/9a)\}}{\left[\frac{(bx)^{2/3}}{b} + \frac{[a(1 - x)]^{2/3}}{a}\right]^{1/2}}$$

and ϕ is the cummulative normal (0, 1) distribution function (Abramovitz and Steger 1972). From this approximation it follows that when $f_0 N > 1$, $x \ll 1$, $f_0 mk \ll 1$, $N/m \gg 1$, and $m > 1$; then

$$\begin{aligned} I_{f_0}(k + 1\ N + 1 - k) &\approx I_{f_0}(k + 1, N) \\ &\approx I_{mf_0}\left(k + 1, \frac{N}{m}\right) \\ &\approx I_{mf_0}\left(k + 1, \frac{N}{m} + 1 - k\right). \end{aligned}$$

This means that instead of having to parametrize by f_0 and N independently, we can create tables parametrized by the product $f_0 N$.

For example, if $f_0 = 0.0001$, $F \geq 10f_0$, and $N = 10^4$, then $K_0 = 1$ and $Prob\ (f \leq f_0 \mid f \leq f_0) = ½[0.6321 + 0.2642] = 0.4481$. If $f_0 = 0.0001$, $F \geq 10f_0$ and $N = 2 \times 10^4$, then $K_0 = 2$ and $Prob\ (f \leq f_0 \mid f \leq f_0) = ⅓[0.8647 + 0.5940 + 0.3233] = 0.5940$. This means that with 2 or fewer observed false alarms out of 20,000 observations, the probability is only 0.5940 that the true false alarm rate is less than 0.0001. It seems that such a policy does not provide very certain answers. Perhaps more observations would be helpful. If $N = 10^5$, then $K_0 = 10$. In this case

$$\begin{aligned} Prob\ (f \leq f_0 \mid \hat{f} \leq f_0) &= \frac{1}{11}[1.0000 + 0.9995 + 0.9972 + 0.9897 \\ &+ 0.9707 + 0.9329 + 0.8699 + 0.7798 + 0.6672 \\ &+ 0.5461 + 0.4170] \\ &= 0.8332. \end{aligned}$$

Thus, with 10 or fewer observed false alarms out of 100,000 observations, the probability is 0.8336 that the true false alarm rate is less than 0.0001. This is certainly better, but, depending on our own requirement for certainty, in our judgment it may not be sure enough.

If we adopt a different policy, we can be more sure about our judgment of the true false alarm rate. Suppose we desire to perform an acceptance test which guarantees that the probability is α that the machine meets specifications. In this case, we adopt the policy that we accept the machine if $f \leq f^*$ where f^* is chosen so that for the fixed probability $\alpha(f^*)$, $Prob\ (f \leq f_0 \mid f \leq f^*) = \alpha(f^*)$. This means to accept if $K \leq K^*$, where $K^* = Nf^*$. Proceeding as before to find K^*, we have

$$\alpha(K^*) = Prob\ (f \leq f_0 \mid \hat{f} \leq f^*)$$

$$= \frac{\int_{f=0}^{f_0} \sum_{k=0}^{K^*} \binom{N}{k} f^k (1 - f)^{N-k} df}{\int_{f=0}^{F} \sum_{k=0}^{K^*} \binom{N}{k} f^k (1 - f)^{N-k} df}$$

$$= \frac{\sum_{k=0}^{K^*} I_{f_0}(k + 1, N + 1 - k)}{\sum_{k=0}^{K^*} I_F(k + 1, N + 1 - k)}$$

$$= \sum_{k=0}^{K^*} \sum_{i=k+1}^{\infty} e^{-K_0} K_0^i / i! \Big/ \sum_{k=0}^{K^*} \sum_{i=k+1}^{\infty} e^{-K_0} K_0^i / i!.$$

Then if $f_0 = 0.001$, $F \geq 10f_0$, $N = 10^5$, and $K^* = 8$, there results

$$\begin{aligned} \alpha\ (K^*) &= \frac{1}{9}[1.000 + 0.9995 + 0.9972 + 0.9897 + 0.9707 \\ &+ 0.9329 + 0.8699 + 0.7798 + 0.6672] \\ &= 0.9119. \end{aligned}$$

So if $f \leq 8/10^5$, the probability will be 0.9119 that the true false alarm rate is less 0.0001.

In summary, we have obtained that $Prob\ (f \leq f_0$ and $K \leq K^*) = 1/N + 1\ \Sigma_{k=0}^{K^*} I_{f_0}\ (k + 1, N + 1 - k)$ and $Prob\ (K \leq K^*) = K^* + 1/N + 1$. Since $Prob\ (f)$ is uniform, $Prob\ (f \leq f_0) = f_0$. These three probabilities determine the missed acceptance rate $Prob$

$(f \leqslant f_0 \mid K > K^*)$, the false acceptance rate *Prob* $(K \leqslant K^* \mid f > f_0)$, the error rate *Prob* $(f \leqslant f_0$ and $K > K^*)$ + *Prob* $(f > f_0$ and $K \leqslant K^*)$, the identification accuracy *Prob* $(f \leqslant f_0$ and $K \leqslant K^*)$ + *Prob* $(f > f_0$ and $K > K^*)$, the acceptance capture rate *Prob* $(K \leqslant K^* \mid f \leqslant f_0)$, and the capture certainty rate $P(f \leqslant f_0 \mid K \leqslant K^*)$. Thus, the complete operating characteristics can be determined of the acceptance policy we have discussed. Tables 1–4 show these relationships for $f_0N = 5$ and $f_0N = 10$. These calculations use the tables of the cummulative Poisson distribution and assume that $F \geqslant 10f_0$.

3 Balancing the Acceptance Test

From the point of view of the buyer of a recognition or inspection machine, one wants to be very sure that the machine being bought meets the specifications. This corresponds to an acceptance test using a small value of K^* and a consequent small false acceptance rate. But as can be seen from Tables 2 and 4, a small false acceptance rate means that the acceptance capture rate (the fraction of machines which are accepted given that they do meet specification) is small. Obviously, the seller of a recognition or inspection machine wants to be sure that the machines which do meet specification are in fact accepted by the buyer. To have a high acceptance capture rate, the seller wants K^* to be large.

If the buyer and seller balance their own self-interests exactly in a middle compromise, the operating point chosen for the acceptance test will be the one for which the false acceptance rate (which the buyer wants to be small) equals the missed acceptance rate (which the seller wants to be small).

In this case

$$Prob\ (K > K^* \mid f \leqslant f_0) = Prob\ (f > f_0 \mid K \leqslant K^*).$$

Since

$$Prob\ (K > K^* \mid f \leqslant f_0) = 1 - Prob\ (K \leqslant K^* \mid f \leqslant f_0)$$

and

$$Prob\ (f > f_0 \mid K \leqslant K^*) = 1 - Prob\ (f \leqslant f_0 \mid K \leqslant K^*)$$

the equality of false acceptance rate and missed acceptance rate implies

Table 2. The missed acceptance rate, the false acceptance rate, and the acceptance capture rate for varying values of K^* when $f_0N = 5$

K^*	Missed Acceptance Rate *Prob* $(K > K^* \mid f \leqslant f_0)$	False Acceptance Rate *Prob* $(f > f_0 \mid K \leqslant K^*)$	Acceptance Capture Rate *Prob* $(K \leqslant K^* \mid f \leqslant f_0)$
00	.8013	.0067	.1987
01	.6094	.0236	.3906
02	.4344	.0573	.5656
03	.2874	.1092	.7126
04	.1755	.1755	.8245
05	.0987	.2489	.8012
06	.0511	.3222	.9489
07	.0244	.3903	.9756
08	.0108	.4504	.9892
09	.0044	.5022	.9956
10	.0017	.5462	.9983
11	.0006	.5836	.9994
12	.0082	.6155	.9998

Table 1. $(N + 1)$ times *Prob* $(f \leqslant f_0$ and $K \leqslant K^*)$, *Prob* $(f \leqslant f_0$ and $K > K^*)$, and *Prob* $(f > f_0$ and $K \leqslant K^*)$ for $f_0N = 5$ for varying values of K^*, the maximum number of permitted false alarms the machine may have and still be accepted

K^*	$(N + 1)$ *Prob* $(f \leqslant f_0$ and $K \leqslant K^*)$	$(N + 1)$ *Prob* $(f \leqslant f_0$ and $K > K^*)$	$(N + 1)$ *Prob* $(f > f_0$ and $K \geqslant K^*)$
00	0.9933	4.0067	0.0067
01	1.9529	3.0471	0.0471
02	2.8282	2.1718	0.1718
03	3.5632	1.4368	0.4368
04	4.1227	0.8773	0.8773
05	4.5067	0.4933	1.4933
06	4.7445	0.2555	2.2555
07	4.8779	0.1221	3.1221
08	4.9460	0.0540	4.0540
09	4.9778	0.0222	5.0222
10	4.9915	0.0085	6.0085
11	4.9970	0.0030	7.0030
12	4.9990	0.0010	8.0010

Table 3. $(N + 1)$ times *Prob* $(f \leqslant f_0$ and $K \leqslant K^*)$, *Prob* $(f \leqslant f_0$ and $K > K^*)$, and *Prob* $(f > f_0$ and $K \leqslant K^*)$ for $f_0N = 10$ for varying values of K^*, the maximum number of permitted false alarms the machine may have and still be accepted

K^*	$(N + 1)$ *Prob* $(f \leqslant f_0$ and $K \leqslant K^*)$	$(N + 1)$ *Prob* $(f \leqslant f_0$ and $K > K^*)$	$(N + 1)$ *Prob* $(f > f_0$ and $K \geqslant K^*)$
00	1.0000	9.0000	0.0000
01	1.9995	8.0005	0.0005
02	2.9967	7.0033	0.0033
03	3.9864	6.0136	0.0136
04	4.9571	5.0429	0.0429
05	5.8892	4.1108	0.1108
06	6.7591	3.2409	0.2409
07	7.5389	2.4611	0.4611
08	8.2061	1.7939	0.7939
09	8.7482	1.2518	1.2518
10	9.1652	0.8548	1.8548
11	9.4684	0.5316	2.5316
12	9.6768	0.3232	3.3232

Table 4. The missed acceptance rate, the false acceptance rate, and the acceptance capture rate for varying values of K^* when $f_0N = 10$

K^*	Missed Acceptance Rate *Prob* $(K > K^* \mid f \leq f_0)$	False Acceptance Rate *Prob* $(f > f_0 \mid K \leq K^*)$	Acceptance Capture Rate *Prob* $(K \leq K^* \mid f \leq f_0)$
00	0.9000	0.0000	0.1000
01	0.8000	0.0003	0.2000
02	0.7003	0.0011	0.2997
03	0.6014	0.0034	0.3986
04	0.5043	0.0086	0.4957
05	0.4111	0.0185	0.5889
06	0.3241	0.0344	0.6759
07	0.2461	0.0576	0.7539
08	0.1794	0.0882	0.8206
09	0.1252	0.1252	0.8748
10	0.0835	0.1668	0.9165
11	0.0532	0.2110	0.9467
12	0.0323	0.2556	0.9677

$$Prob\ (K \leq K^* \mid f \leq f_0) = Prob\ (f \leq f_0 \mid K \leq K^*).$$

But

$$Prob\ (K \leq K^* \mid f \leq f_0) = \frac{Prob\ (f \leq f_0 \mid K \leq K^*)\ Prob\ (K \leq K^*)}{Prob\ (f \leq f_0)}.$$

Hence the equality of the false acceptance rate and missed acceptance rate implies *Prob* $(f \leq f_0) =$ *Prob* $(K \leq K^*)$. Using the uniform distribution assumption on f, there results $f_0 = (K^* + 1)/(N + 1)$. From this we obtain $K^* = f_0(N + 1) - 1$. For N large this can be simplified to $K^* = f_0N - 1$. Choosing an operating point with a value of K^* less than $f_0N - 1$ favors the buyer and choosing an operating point with a value of K^* greater than $f_0N - 1$ favors the seller.

The last issue we address is the relationship between the number of trials for the acceptance test and the false acceptance rate at the operating point which has the false acceptance rate equal to the missed acceptance rate. Now the false acceptance rate is given by

$$Prob\ (f > f_0 \mid K \leq K^*) = 1 - \frac{1}{K^* + 1} \sum_{k=0}^{K^*} \sum_{i=k+1}^{\infty} e^{-K_0} K_0^i / i!.$$

At the desired operating point $K^* = f_0N - 1 = K_0 - 1$. To illustrate the relationship, we just compute $1/Nf_0 \sum_{k=0}^{Nf_0 - 1} \sum_{i=k+1}^{\infty} e^{-Nf_0} (Nf_0)^i/i!$ as a function of N. Since N occurs everywhere with f_0, we can compute the false acceptance rate as a function of $K_0 = Nf_0$ and obtain a little more generality. Table 5 shows that with $Nf_0 = 5$, the false acceptance rate is 0.1755. With $Nf_0 = 10$ the false acceptance rate is 0.1251. And with $Nf_0 = 20$ the false acceptance rate is 0.0888. This suggests that, for the acceptance test to have certainty greater than 90%, $Nf_0 \geq 15$.

Table 5. The false acceptance rate as a function of $K_0 = Nf_0$ where $K^* = Nf_0 - 1$ is chosen to equalize the false acceptance and missed acceptance rate

$K_0 = Nf_0$	$1 - 1/K_0 \sum_{k=0}^{K_0-1} \sum_{i=k+1}^{\infty} e^{-K_0} K_0^i/i!$
01	0.3679
02	0.2706
03	0.2240
04	0.1954
05	0.1755
06	0.1607
07	0.1490
08	0.1395
09	0.1318
10	0.1251
15	0.1025
20	0.0888
30	0.0729
40	0.0629

4 Tables

In order for the discussion of this note to be most useful, we have used the incomplete Beta ratio approximation given by Abramovitz and Steger (1972) to calculate the probability of missed acceptance and false acceptance rates for $f_0N = 1$ to 15 stepping by 1 and 20 to 50 stepping by 5. The prior support F can be set to f_0, $2f_0$, $3f_0$, $4f_0$, $5f_0$, and $10f_0$. The value of F does not influence the missed acceptance rate but it does influence the false acceptance rate. It can be seen from the tables that F has most effect for smaller f_0N but even there the effect is not major. The machine acceptance test is to accept if the number of defects is equal to or less than K^*. In constructing the tables, the numerical value of N, the sample size, was 10,000. For sample sizes as small as $N = 100$, the tables are accurate to at least two decimal places. For sample sizes larger than 10,000, the tables are accurate to four decimal places.

5 Concluding Discussion

With reference to performance assessment of decision rules, Highleyman (1962), Fukunaga (1972), Duda and Hunt (1973), and Devijver and Kittler (1982) all treat the problem as one of determining 95% confidence intervals for the true error rate given the observed error rate and the number of samples. If the required error rate lies outside the confidence interval, then the unit being tested would fail the acceptance test. This kind of a test, to be appropriate, would require a one-sided confidence interval instead of the two-sided confidence interval each discusses. However, even in the case of the one-sided confidence interval, there are important differences between what they calculate and what we do. In our notation, the one-sided confidence interval $[0, f_0]$ they should compute is associated with the probability *Prob* $(f \leq f_0 \mid f = f_0)$, a conditional probability on the observed error rate given that the true error rate is f_0. The conditional probability which is the more meaningful and useful conditional probability and the one we compute is *Prob* $(f \leq f_0 \mid f \leq f^*)$, a conditional probability on the true error rate f being not greater than f_0 given that the observed error rate f is not greater than f^*. Further, we choose the f^* to depend on f_0 and N such that the two types of error probabilities are equal: *Prob* $(f > f_0 \mid f \leq f^*)$ = *Prob* $(f \leq f_0 \mid f > f^*)$.

Concerns about performance assessment can be found in the statistical literature on reliability and the statistical literature on Monte Carlo tests (cf., for example, Minott 1979, Jockel, 1986). The contribution in this paper is the publishing of a convenient set of tables specialized for small error rates for a test which equalizes the type I and type II errors.

Appendix: Tables

The following tables give the missed acceptance rate and false acceptance rate of the performance assessment test as a function of the product f_0N, where f_0 is the specified error rate and N is the sample size, where K^*, is the test threshold, and where $[0, F]$ is the interval of support for the prior probability of the unknown error f. If K is the number of test items in error from the sample of N, then the missed acceptance rate is *Prob* $(f < f_0 \mid K > K^*)$ and the false acceptance rate is *Prob* $(f > f_0 \mid K < K^*)$. For example, if $f_0N = 8$, $F = 5f_0$, and the test threshold is $K^* = 9$, then the missed acceptance rate, $P(f < f_0 \mid K > K^*) = 0.0885$ and the false acceptance rate, $P(f > f_0 \mid K < K^*) = 0.2425$.

f0*N = 1

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.3694	0.2728	0.3373	0.3578	0.3650	0.3693
1	0.1082	0.3893	0.4915	0.5285	0.5434	0.5539
2	0.0276	0.4542	0.5827	0.6336	0.6563	0.6754
3	0.0076	0.4841	0.6299	0.6918	0.7216	0.7510
4	0.0034	0.4958	0.6522	0.7224	0.7584	0.7989
5	0.0026	0.4997	0.6618	0.7378	0.7787	0.8307
6	0.0025	0.5009	0.6656	0.7452	0.7898	0.8524

f0*N = 2

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.5664	0.1168	0.1305	0.1324	0.1327	0.1328
1	0.2698	0.2279	0.2624	0.2684	0.2695	0.2698
2	0.1092	0.3288	0.3896	0.4026	0.4054	0.4061
3	0.0382	0.4027	0.4895	0.5118	0.5174	0.5191
4	0.0117	0.4495	0.5592	0.5917	0.6012	0.6046
5	0.0032	0.4760	0.6043	0.6472	0.6615	0.6677
6	0.0007	0.4895	0.6320	0.6847	0.7044	0.7144
7	0.0001	0.4958	0.6483	0.7095	0.7348	0.7500

f0*N = 3

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.6828	0.0459	0.0483	0.0484	0.0484	0.0484
1	0.4154	0.1142	0.1224	0.1230	0.1230	0.1230
2	0.2233	0.2016	0.2212	0.2230	0.2232	0.2232
3	0.1062	0.2884	0.3247	0.3291	0.3295	0.3296
4	0.0450	0.3610	0.4172	0.4257	0.4268	0.4269
5	0.0170	0.4146	0.4916	0.5059	0.5081	0.5085
6	0.0057	0.4508	0.5477	0.5691	0.5731	0.5738
7	0.0017	0.4733	0.5881	0.6177	0.6242	0.6256
8	0.0003	0.4863	0.6163	0.6545	0.6642	0.6667

f0*N = 4

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.7545	0.0176	0.0180	0.0180	0.0180	0.0180
1	0.5271	0.0525	0.0541	0.0542	0.0542	0.0542
2	0.3364	0.1100	0.1149	0.1151	0.1151	0.1151
3	0.1949	0.1826	0.1942	0.1947	0.1948	0.1948
4	0.1023	0.2583	0.2802	0.2817	0.2818	0.2818
5	0.0489	0.3267	0.3625	0.3656	0.3658	0.3658
6	0.0213	0.3823	0.4345	0.4402	0.4406	0.4407
7	0.0085	0.4239	0.4937	0.5032	0.5041	0.5042
8	0.0031	0.4531	0.5405	0.5550	0.5567	0.5569
9	0.0010	0.4724	0.5766	0.5971	0.6000	0.6003

f0*N = 5

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.8014	0.0068	0.0069	0.0069	0.0069	0.0069
1	0.6094	0.0231	0.0234	0.0234	0.0234	0.0234
2	0.4341	0.0557	0.0568	0.0568	0.0568	0.0568
3	0.2870	0.1055	0.1086	0.1086	0.1086	0.1086
4	0.1751	0.1679	0.1748	0.1750	0.1750	0.1750
5	0.0985	0.2347	0.2481	0.2486	0.2486	0.2486
6	0.0511	0.2981	0.3210	0.3221	0.3221	0.3221
7	0.0245	0.3532	0.3880	0.3902	0.3902	0.3902
8	0.0109	0.3975	0.4463	0.4502	0.4504	0.4504
9	0.0045	0.4311	0.4953	0.5018	0.5022	0.5022
10	0.0017	0.4555	0.5354	0.5454	0.5461	0.5462

f0*N = 6

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.8338	0.0027	0.0027	0.0027	0.0027	0.0027
1	0.6700	0.0099	0.0100	0.0100	0.0100	0.0100
2	0.5136	0.0269	0.0271	0.0271	0.0271	0.0271
3	0.3720	0.0572	0.0579	0.0579	0.0579	0.0579
4	0.2527	0.1012	0.1032	0.1032	0.1032	0.1032
5	0.1604	0.1559	0.1602	0.1603	0.1603	0.1603
6	0.0949	0.2155	0.2239	0.2241	0.2241	0.2241
7	0.0523	0.2741	0.2888	0.2892	0.2892	0.2892
8	0.0269	0.3273	0.3504	0.3512	0.3512	0.3512
9	0.0130	0.3725	0.4061	0.4077	0.4077	0.4077
10	0.0058	0.4089	0.4548	0.4576	0.4577	0.4577
11	0.0025	0.4369	0.4965	0.5010	0.5012	0.5012

f0*N = 7

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.8573	0.0011	0.0011	0.0011	0.0011	0.0011
1	0.7155	0.0042	0.0042	0.0042	0.0042	0.0042
2	0.5769	0.0126	0.0127	0.0127	0.0127	0.0127
3	0.4456	0.0296	0.0298	0.0298	0.0298	0.0298
4	0.3274	0.0578	0.0582	0.0582	0.0582	0.0582
5	0.2274	0.0973	0.0985	0.0985	0.0985	0.0985
6	0.1488	0.1460	0.1487	0.1487	0.1487	0.1487
7	0.0915	0.1996	0.2050	0.2050	0.2050	0.2050
8	0.0529	0.2537	0.2632	0.2633	0.2633	0.2633
9	0.0288	0.3045	0.3198	0.3201	0.3201	0.3201
10	0.0148	0.3493	0.3723	0.3730	0.3730	0.3730
11	0.0071	0.3871	0.4196	0.4207	0.4208	0.4208
12	0.0033	0.4177	0.4612	0.4632	0.4632	0.4632

f0*N = 8

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.8751	0.0004	0.0004	0.0004	0.0004	0.0004
1	0.7505	0.0018	0.0018	0.0018	0.0018	0.0018
2	0.6272	0.0058	0.0058	0.0058	0.0058	0.0058
3	0.5075	0.0148	0.0149	0.0149	0.0149	0.0149
4	0.3949	0.0316	0.0317	0.0317	0.0317	0.0317
5	0.2937	0.0578	0.0581	0.0581	0.0581	0.0581
6	0.2078	0.0937	0.0945	0.0945	0.0945	0.0945
7	0.1394	0.1376	0.1393	0.1393	0.1393	0.1393
8	0.0885	0.1863	0.1897	0.1897	0.1897	0.1897
9	0.0532	0.2363	0.2424	0.2425	0.2425	0.2425
10	0.0302	0.2844	0.2945	0.2946	0.2946	0.2946
11	0.0163	0.3282	0.3439	0.3441	0.3441	0.3441
12	0.0083	0.3664	0.3892	0.3897	0.3897	0.3897
13	0.0040	0.3986	0.4300	0.4308	0.4308	0.4308

f0*N = 9

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.8889	0.0002	0.0002	0.0002	0.0002	0.0002
1	0.7780	0.0008	0.0008	0.0008	0.0008	0.0008
2	0.6676	0.0026	0.0026	0.0026	0.0026	0.0026
3	0.5588	0.0073	0.0073	0.0073	0.0073	0.0073
4	0.4538	0.0167	0.0167	0.0167	0.0167	0.0167
5	0.3555	0.0330	0.0331	0.0331	0.0331	0.0331
6	0.2673	0.0576	0.0578	0.0578	0.0578	0.0578
7	0.1921	0.0905	0.0910	0.0910	0.0910	0.0910
8	0.1316	0.1304	0.1315	0.1315	0.1315	0.1315
9	0.0858	0.1749	0.1771	0.1771	0.1771	0.1771
10	0.0532	0.2212	0.2252	0.2253	0.2253	0.2253
11	0.0314	0.2666	0.2734	0.2734	0.2734	0.2734
12	0.0176	0.3090	0.3197	0.3198	0.3198	0.3198
13	0.0094	0.3470	0.3629	0.3631	0.3631	0.3631
14	0.0048	0.3800	0.4025	0.4028	0.4028	0.4028

f0*N = 10

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9000	0.0001	0.0001	0.0001	0.0001	0.0001
1	0.8001	0.0003	0.0003	0.0003	0.0003	0.0003
2	0.7004	0.0012	0.0012	0.0012	0.0012	0.0012
3	0.6014	0.0035	0.0035	0.0035	0.0035	0.0035
4	0.5044	0.0086	0.0086	0.0086	0.0086	0.0086
5	0.4110	0.0183	0.0183	0.0183	0.0183	0.0183
6	0.3240	0.0341	0.0342	0.0342	0.0342	0.0342
7	0.2459	0.0572	0.0573	0.0573	0.0573	0.0573
8	0.1792	0.0876	0.0879	0.0879	0.0879	0.0879
9	0.1250	0.1242	0.1249	0.1249	0.1249	0.1249
10	0.0833	0.1651	0.1666	0.1666	0.1666	0.1666
11	0.0530	0.2081	0.2108	0.2108	0.2108	0.2108
12	0.0322	0.2509	0.2555	0.2555	0.2555	0.2555
13	0.0187	0.2917	0.2990	0.2990	0.2990	0.2990
14	0.0104	0.3290	0.3401	0.3402	0.3402	0.3402
15	0.0055	0.3622	0.3782	0.3784	0.3784	0.3874

f0*N = 11

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9091	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8182	0.0001	0.0001	0.0001	0.0001	0.0001
2	0.7274	0.0005	0.0005	0.0005	0.0005	0.0005
3	0.6370	0.0017	0.0017	0.0017	0.0017	0.0017
4	0.5475	0.0043	0.0043	0.0043	0.0043	0.0043
5	0.4600	0.0098	0.0098	0.0098	0.0098	0.0098
6	0.3762	0.0196	0.0196	0.0196	0.0196	0.0196
7	0.2982	0.0349	0.0350	0.0350	0.0350	0.0350
8	0.2284	0.0567	0.0568	0.0568	0.0568	0.0568
9	0.1684	0.0849	0.0851	0.0851	0.0851	0.0851
10	0.1193	0.1187	0.1192	0.1192	0.1192	0.1192
11	0.0810	0.1566	0.1575	0.1575	0.1575	0.1575
12	0.0528	0.1966	0.1984	0.1984	0.1984	0.1984
13	0.0329	0.2370	0.2401	0.2401	0.2401	0.2401
14	0.0197	0.2760	0.2810	0.2810	0.2810	0.2810
15	0.0113	0.3124	0.3202	0.3202	0.3202	0.3202
16	0.0062	0.3455	0.3568	0.3569	0.3569	0.3569

f0*N = 12

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9167	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8334	0.0001	0.0001	0.0001	0.0001	0.0001
2	0.7501	0.0002	0.0002	0.0002	0.0002	0.0002
3	0.6670	0.0008	0.0008	0.0008	0.0008	0.0008
4	0.5843	0.0022	0.0022	0.0022	0.0022	0.0022
5	0.5026	0.0052	0.0052	0.0052	0.0052	0.0052
6	0.4231	0.0110	0.0110	0.0110	0.0110	0.0110
7	0.3472	0.0207	0.0207	0.0207	0.0207	0.0207
8	0.2767	0.0355	0.0356	0.0356	0.0356	0.0356
9	0.2136	0.0561	0.0562	0.0562	0.0562	0.0562
10	0.1591	0.0825	0.0826	0.0826	0.0826	0.0826
11	0.1143	0.1138	0.1142	0.1142	0.1142	0.1142
12	0.0789	0.1491	0.1497	0.1497	0.1497	0.1497
13	0.0524	0.1865	0.1877	0.1877	0.1877	0.1877
14	0.0335	0.2246	0.2267	0.2267	0.2267	0.2267
15	0.0205	0.2619	0.2653	0.2653	0.2653	0.2653
16	0.0121	0.2972	0.3026	0.3026	0.3026	0.3026
17	0.0069	0.3298	0.3378	0.3378	0.3378	0.3378

f0*N = 13

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9231	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8462	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.7693	0.0001	0.0001	0.0001	0.0001	0.0001
3	0.6924	0.0004	0.0004	0.0004	0.0004	0.0004
4	0.6158	0.0011	0.0011	0.0011	0.0011	0.0011
5	0.5397	0.0027	0.0027	0.0027	0.0027	0.0027
6	0.4648	0.0060	0.0060	0.0060	0.0060	0.0060
7	0.3920	0.0119	0.0120	0.0120	0.0120	0.0120
8	0.3227	0.0216	0.0216	0.0216	0.0216	0.0216
9	0.2585	0.0360	0.0360	0.0360	0.0360	0.0360
10	0.2009	0.0555	0.0556	0.0556	0.0556	0.0556
11	0.1511	0.0802	0.0803	0.0803	0.0803	0.0803
12	0.1098	0.1095	0.1097	0.1097	0.1097	0.1097
13	0.0770	0.1424	0.1428	0.1428	0.1428	0.1428
14	0.0520	0.1776	0.1784	0.1784	0.1784	0.1784

			f0*N = 13			
		FALSE	FALSE	FALSE	FALSE	FALSE
F = L*f0	L =	2	3	4	5	10
KSTAR	MISSED					
15	0.0339	0.2135	0.2150	0.2150	0.2150	0.2150
16	0.0213	0.2491	0.2515	0.2515	0.2515	0.2515
17	0.0129	0.2832	0.2870	0.2870	0.2870	0.2870
18	0.0075	0.3151	0.3208	0.3208	0.3208	0.3208

			f0*N = 14			
		FALSE	FALSE	FALSE	FALSE	FALSE
F = L*f0	L =	2	3	4	5	10
KSTAR	MISSED					
0	0.9286	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8572	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.7857	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.7144	0.0002	0.0002	0.0002	0.0002	0.0002
4	0.6431	0.0005	0.0005	0.0005	0.0005	0.0005
5	0.5721	0.0014	0.0014	0.0014	0.0014	0.0014
6	0.5016	0.0032	0.0032	0.0032	0.0032	0.0032
7	0.4325	0.0067	0.0067	0.0067	0.0067	0.0067
8	0.3655	0.0128	0.0128	0.0128	0.0128	0.0128
9	0.3018	0.0224	0.0224	0.0224	0.0224	0.0224
10	0.2429	0.0363	0.0363	0.0363	0.0363	0.0363
11	0.1900	0.0549	0.0549	0.0549	0.0549	0.0549
12	0.1442	0.0782	0.0782	0.0782	0.0782	0.0782
13	0.1059	0.1057	0.1058	0.1058	0.1058	0.1058
14	0.0752	0.1365	0.1368	0.1368	0.1368	0.1368
15	0.0516	0.1695	0.1701	0.1701	0.1701	0.1701
16	0.0342	0.2036	0.2046	0.2046	0.2046	0.2046
17	0.0219	0.2376	0.2392	0.2392	0.2392	0.2392
18	0.0135	0.2704	0.2731	0.2731	0.2731	0.2731
19	0.0081	0.3015	0.3056	0.3056	0.3056	0.3056

			f0*N = 15			
		FALSE	FALSE	FALSE	FALSE	FALSE
F = L*f0	L =	2	3	4	5	10
KSTAR	MISSED					
0	0.9333	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8667	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.7334	0.0001	0.0001	0.0001	0.0001	0.0001
4	0.6668	0.0002	0.0002	0.0002	0.0002	0.0002
5	0.6003	0.0007	0.0007	0.0007	0.0007	0.0007
6	0.5342	0.0017	0.0017	0.0017	0.0017	0.0017
7	0.4687	0.0037	0.0037	0.0037	0.0037	0.0037
8	0.4045	0.0074	0.0074	0.0074	0.0074	0.0074
9	0.3425	0.0137	0.0137	0.0137	0.0137	0.0137
10	0.2837	0.0231	0.0231	0.0231	0.0231	0.0231
11	0.2293	0.0365	0.0365	0.0365	0.0365	0.0365
12	0.1804	0.0543	0.0543	0.0543	0.0543	0.0543
13	0.1380	0.0763	0.0763	0.0763	0.0763	0.0763
14	0.1023	0.1022	0.1023	0.1023	0.1023	0.1023
15	0.0736	0.1312	0.1314	0.1314	0.1314	0.1314
16	0.0512	0.1624	0.1627	0.1627	0.1627	0.1627
17	0.0345	0.1946	0.1953	0.1953	0.1953	0.1953
18	0.0225	0.2271	0.2282	0.2282	0.2282	0.2282
19	0.0142	0.2587	0.2605	0.2605	0.2605	0.2605
20	0.0086	0.2890	0.2918	0.2918	0.2918	0.2918

f0*N = 20

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9500	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8500	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.7500	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.7000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.6501	0.0001	0.0001	0.0001	0.0001	0.0001
7	0.6001	0.0001	0.0001	0.0001	0.0001	0.0001
8	0.5502	0.0004	0.0004	0.0004	0.0004	0.0004
9	0.5005	0.0008	0.0008	0.0008	0.0008	0.0008
10	0.4510	0.0017	0.0017	0.0017	0.0017	0.0017
11	0.4021	0.0034	0.0034	0.0034	0.0034	0.0034
12	0.3540	0.0061	0.0061	0.0061	0.0061	0.0061
13	0.3073	0.0104	0.0104	0.0104	0.0104	0.0104
14	0.2625	0.0166	0.0166	0.0166	0.0166	0.0166
15	0.2203	0.0253	0.0253	0.0253	0.0253	0.0253
16	0.1814	0.0368	0.0368	0.0368	0.0368	0.0368
17	0.1462	0.0512	0.0512	0.0512	0.0512	0.0512
18	0.1152	0.0686	0.0686	0.0686	0.0686	0.0686
19	0.0888	0.0887	0.0887	0.0887	0.0887	0.0887
20	0.0667	0.1110	0.1111	0.1111	0.1111	0.1111
21	0.0489	0.1352	0.1353	0.1353	0.1353	0.1353
22	0.0349	0.1606	0.1607	0.1607	0.1607	0.1607
23	0.0243	0.1867	0.1869	0.1869	0.1869	0.1869
24	0.0165	0.2128	0.2131	0.2131	0.2131	0.2131
25	0.0109	0.2386	0.2391	0.2391	0.2391	0.2391

f0*N = 25

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9600	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9200	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8800	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.8400	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.7600	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.7200	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.6800	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.6400	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.6001	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.5601	0.0001	0.0001	0.0001	0.0001	0.0001
11	0.5201	0.0002	0.0002	0.0002	0.0002	0.0002
12	0.4803	0.0004	0.0004	0.0004	0.0004	0.0004
13	0.4405	0.0009	0.0009	0.0009	0.0009	0.0009
14	0.4010	0.0016	0.0016	0.0016	0.0016	0.0016
15	0.3619	0.0029	0.0029	0.0029	0.0029	0.0029
16	0.3234	0.0049	0.0049	0.0049	0.0049	0.0049
17	0.2858	0.0080	0.0080	0.0080	0.0080	0.0080
18	0.2495	0.0124	0.0124	0.0124	0.0124	0.0124
19	0.2148	0.0184	0.0184	0.0184	0.0184	0.0184
20	0.1822	0.0264	0.0264	0.0264	0.0264	0.0264
21	0.1521	0.0364	0.0364	0.0364	0.0364	0.0364
22	0.1248	0.0486	0.0486	0.0486	0.0486	0.0486
23	0.1005	0.0629	0.0629	0.0629	0.0629	0.0629
24	0.0794	0.0794	0.0794	0.0794	0.0794	0.0794
25	0.0616	0.0976	0.0976	0.0976	0.0976	0.0976
26	0.0467	0.1172	0.1173	0.1173	0.1173	0.1173
27	0.0348	0.1381	0.1381	0.1381	0.1381	0.1381
28	0.0253	0.1596	0.1597	0.1597	0.1597	0.1597
29	0.0180	0.1816	0.1816	0.1816	0.1816	0.1816
30	0.0126	0.2035	0.2036	0.2036	0.2036	0.2036

			f0*N = 30			
		FALSE	FALSE	FALSE	FALSE	FALSE
F = L*f0	L =	2	3	4	5	10
KSTAR	MISSED					
0	0.9667	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9333	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.8667	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8333	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.7667	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.7334	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.7000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.6334	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.6000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.5667	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.5334	0.0001	0.0001	0.0001	0.0001	0.0001
14	0.5001	0.0001	0.0001	0.0001	0.0001	0.0001
15	0.4668	0.0002	0.0002	0.0002	0.0002	0.0002
16	0.4336	0.0004	0.0004	0.0004	0.0004	0.0004
17	0.4006	0.0008	0.0008	0.0008	0.0008	0.0008
18	0.3676	0.0015	0.0015	0.0015	0.0015	0.0015
19	0.3350	0.0025	0.0025	0.0025	0.0025	0.0025
20	0.3029	0.0040	0.0040	0.0040	0.0040	0.0040
21	0.2714	0.0063	0.0063	0.0063	0.0063	0.0063
22	0.2407	0.0095	0.0095	0.0095	0.0095	0.0095
23	0.2112	0.0139	0.0139	0.0139	0.0139	0.0139
24	0.1831	0.0196	0.0196	0.0196	0.0196	0.0196
25	0.1566	0.0268	0.0268	0.0268	0.0268	0.0268
26	0.1322	0.0357	0.0357	0.0357	0.0357	0.0357
27	0.1099	0.0463	0.0463	0.0463	0.0463	0.0463
28	0.0900	0.0586	0.0586	0.0586	0.0586	0.0586
29	0.0726	0.0725	0.0725	0.0725	0.0725	0.0725
30	0.0575	0.0878	0.0878	0.0878	0.0878	0.0878
31	0.0448	0.1044	0.1044	0.1044	0.1044	0.1044
32	0.0343	0.1220	0.1220	0.1220	0.1220	0.1220
33	0.0258	0.1403	0.1403	0.1403	0.1403	0.1403
34	0.0190	0.1591	0.1591	0.1591	0.1591	0.1591
35	0.0138	0.1781	0.1781	0.1781	0.1781	0.1781

			f0*N = 35			
		FALSE	FALSE	FALSE	FALSE	FALSE
F = L*f0	L =	2	3	4	5	10
KSTAR	MISSED					
0	0.9714	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9429	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9143	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.8857	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8572	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.8286	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.7715	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.7429	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.7143	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.6857	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.6572	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.6286	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.6000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.5715	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.5429	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.5143	0.0000	0.0000	0.0000	0.0000	0.0000
17	0.4858	0.0001	0.0001	0.0001	0.0001	0.0001
18	0.4573	0.0001	0.0001	0.0001	0.0001	0.0001
19	0.4288	0.0002	0.0002	0.0002	0.0002	0.0002
20	0.4003	0.0004	0.0004	0.0004	0.0004	0.0004

f0*N = 35

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
21	0.3720	0.0007	0.0007	0.0007	0.0007	0.0007
22	0.3438	0.0013	0.0013	0.0013	0.0013	0.0013
23	0.3158	0.0021	0.0021	0.0021	0.0021	0.0021
24	0.2881	0.0033	0.0033	0.0033	0.0033	0.0033
25	0.2609	0.0050	0.0050	0.0050	0.0050	0.0050
26	0.2344	0.0074	0.0074	0.0074	0.0074	0.0074
27	0.2086	0.0107	0.0107	0.0107	0.0107	0.0107
28	0.1839	0.0149	0.0149	0.0149	0.0149	0.0149
29	0.1603	0.0203	0.0203	0.0203	0.0203	0.0203
30	0.1382	0.0269	0.0269	0.0269	0.0269	0.0269
31	0.1177	0.0349	0.0349	0.0349	0.0349	0.0349
32	0.0990	0.0443	0.0443	0.0443	0.0443	0.0443
33	0.0821	0.0550	0.0550	0.0550	0.0550	0.0550
34	0.0672	0.0671	0.0671	0.0671	0.0671	0.0671
35	0.0542	0.0804	0.0804	0.0804	0.0804	0.0804
36	0.0430	0.0947	0.0947	0.0947	0.0947	0.0947
37	0.0337	0.1099	0.1099	0.1099	0.1099	0.1099
38	0.0259	0.1258	0.1258	0.1258	0.1258	0.1258
39	0.0197	0.1421	0.1421	0.1421	0.1421	0.1421
40	0.0147	0.1588	0.1588	0.1588	0.1588	0.1588

f0*N = 40

F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9750	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9500	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9250	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8750	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.8500	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.8250	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.7750	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.7500	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.7250	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.7000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.6750	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.6500	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.6250	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.6000	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.5750	0.0000	0.0000	0.0000	0.0000	0.0000
17	0.5500	0.0000	0.0000	0.0000	0.0000	0.0000
18	0.5251	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.5001	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.4751	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.4501	0.0001	0.0001	0.0001	0.0001	0.0001
22	0.4251	0.0001	0.0001	0.0001	0.0001	0.0001
23	0.4002	0.0002	0.0002	0.0002	0.0002	0.0002
24	0.3753	0.0004	0.0004	0.0004	0.0004	0.0004
25	0.3505	0.0007	0.0007	0.0007	0.0007	0.0007
26	0.3258	0.0011	0.0011	0.0011	0.0011	0.0011
27	0.3013	0.0017	0.0017	0.0017	0.0017	0.0017
28	0.2770	0.0027	0.0027	0.0027	0.0027	0.0027
29	0.2531	0.0040	0.0040	0.0040	0.0040	0.0040
30	0.2296	0.0059	0.0059	0.0059	0.0059	0.0059
31	0.2067	0.0083	0.0083	0.0083	0.0083	0.0083
32	0.1846	0.0116	0.0116	0.0116	0.0116	0.0116
33	0.1634	0.0157	0.0157	0.0157	0.0157	0.0157
34	0.1432	0.0207	0.0207	0.0207	0.0207	0.0207

			f0*N = 40			
F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
35	0.1243	0.0269	0.0269	0.0269	0.0269	0.0269
36	0.1066	0.0341	0.0341	0.0341	0.0341	0.0341
37	0.0905	0.0425	0.0425	0.0425	0.0425	0.0425
38	0.0759	0.0521	0.0521	0.0521	0.0521	0.0521
39	0.0629	0.0628	0.0628	0.0628	0.0628	0.0628
40	0.0514	0.0744	0.0744	0.0744	0.0744	0.0744
41	0.0415	0.0870	0.0870	0.0870	0.0870	0.0870
42	0.0330	0.1004	0.1004	0.1004	0.1004	0.1004
43	0.0259	0.1144	0.1144	0.1144	0.1144	0.1144
44	0.0201	0.1289	0.1289	0.1289	0.1289	0.1289
45	0.0153	0.1437	0.1437	0.1437	0.1437	0.1437

			f0*N = 45			
F = L*f0 KSTAR	L = MISSED	FALSE 2	FALSE 3	FALSE 4	FALSE 5	FALSE 10
0	0.9778	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9556	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9333	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9111	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8889	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.8667	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.8445	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.8222	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.8000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.7778	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.7556	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.7334	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.7111	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.6889	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.6445	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.6223	0.0000	0.0000	0.0000	0.0000	0.0000
17	0.6000	0.0000	0.0000	0.0000	0.0000	0.0000
18	0.5778	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.5556	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.5334	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.5112	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.4889	0.0000	0.0000	0.0000	0.0000	0.0000
23	0.4667	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.4445	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.4223	0.0001	0.0001	0.0001	0.0001	0.0001
26	0.4001	0.0001	0.0001	0.0001	0.0001	0.0001
27	0.3780	0.0002	0.0002	0.0002	0.0002	0.0002
28	0.3558	0.0004	0.0004	0.0004	0.0004	0.0004
29	0.3338	0.0006	0.0006	0.0006	0.0006	0.0006
30	0.3118	0.0009	0.0009	0.0009	0.0009	0.0009
31	0.2900	0.0015	0.0015	0.0015	0.0015	0.0015
32	0.2684	0.0022	0.0022	0.0022	0.0022	0.0022
33	0.2470	0.0033	0.0033	0.0033	0.0033	0.0033
34	0.2260	0.0047	0.0047	0.0047	0.0047	0.0047
35	0.2054	0.0066	0.0066	0.0066	0.0066	0.0066
36	0.1853	0.0091	0.0091	0.0091	0.0091	0.0091
37	0.1660	0.0123	0.0123	0.0123	0.0123	0.0123
38	0.1475	0.0162	0.0162	0.0162	0.0162	0.0162
39	0.1298	0.0210	0.0210	0.0210	0.0210	0.0210
40	0.1133	0.0267	0.0267	0.0267	0.0267	0.0267
41	0.0979	0.0333	0.0333	0.0333	0.0333	0.0333
42	0.0837	0.0410	0.0410	0.0410	0.0410	0.0410
43	0.0708	0.0496	0.0496	0.0496	0.0496	0.0496

			f0*N = 45			
		FALSE	FALSE	FALSE	FALSE	FALSE
F = L*f0	L =	2	3	4	5	10
KSTAR	MISSED					
44	0.0593	0.0592	0.0592	0.0592	0.0592	0.0592
45	0.0490	0.0696	0.0696	0.0696	0.0696	0.0696
46	0.0401	0.0808	0.0808	0.0808	0.0808	0.0808
47	0.0324	0.0928	0.0928	0.0928	0.0928	0.0928
48	0.0258	0.1053	0.1053	0.1053	0.1053	0.1053
49	0.0204	0.1182	0.1182	0.1182	0.1182	0.1182
50	0.0158	0.1315	0.1315	0.1315	0.1315	0.1315

References

Abramovitz M, Stegum I (1972) Handbook of Mathematical Functions, Dover, New York

Devijver PA, Kittler J (1982) Pattern Recognition: A Statistical Approach, Prentice Hall, London

Duda RO, Hart PF (1973) Pattern Classification and Scene Analysis, Wiley, New York

Fununaga K (1972) Introduction to Statistical Pattern Recognition, Academic, New York

Highleyman WH (1962) The design and analysis of pattern recognition experiments. Bell System Technical Journal, 41: 723–744

Jockel KH (1986) Finite sample properties and asymptotic efficiency of Monte Carlo tests. The Annals of Statics, 14: a366–a347

Johnson N, Kotz S (1969) Discrete Distributions. Hougton Mifflin, Boston

Johnson N, Kotz S (1970) Continuous Univariate Distribution—2. Houghton Mifflin, Boston

Marriott FHC (1979) Barmarks Monte Carlo tests: How many simulations? Applied Statistics, 28: 75–77

Pearson ES, Hartley HO (1958) Biometrical Tables for Statisticians, Vol. 1 (2nd Edition) Cambridge, London

Pearson K (1968) Tables of Incomplete Beta Function (2nd Edition) Cambridge, London