Pattern Recognition Vol. 16, No. 1, pp. 23–29, 1983.
Printed in Great Britain.

0031–3203/83/010023–07 $03.00/0
Pergamon Press Ltd.

PEAK NOISE REMOVAL BY A FACET MODEL*

YOSHIFUMI YASUOKA† and ROBERT M. HARALICK‡

Spatial Data Analysis Laboratory, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, U.S.A.

(*Received* 21 *August* 1981; *in revised form* 2 *February* 1982; *received for publication* 25 *February* 1982)

Abstract—A peak noise removal method based on local gray tone statistics from the facet model is introduced. Each pixel in an image is statistically tested to determine whether it belongs to the same gray tone intensity surface as its neighborhood pixels. If its gray tone is outside the 95 per cent confidence interval estimated from the neighborhood gray tones, it is judged as peak noise and its value is replaced by an average of the gray tone values of the neighborhood pixels. In order to estimate the local gray tone statistics, an assumption is made that the neighborhood region is described by a linear or quadratic facet surface model. It is also shown that this method can be successfully applied to scan line noise removal by using a one-dimensional (horizontal or vertical) neighborhood.

I. INTRODUCTION

Noise cleaning is one of the basic problems in image processing. There have been many papers on noise cleaning or smoothing of image data (see Rosenfeld and Yak[1]). One of the difficulties in this problem is that we need different algorithms or methods for different kinds of noise. A box type smoothing or a nearest neighborhood type smoothing removes Gaussian noise, however it sometimes diffuses noise around the neighborhood if it is applied to peak noise or scan line noise without care. Median filtering[2] can remove this kind of noise considerably better, but it also tends to smooth out or blur fine structures in an image and to cause image degradation.

This paper discusses a method for detecting and removing peak noise based on the local gray tone statistics of the neighborhood region. Peak noise, which is usually added to image data in the imaging process or digitizing process, is considered to be a pixel whose gray tone differs sufficiently from the neighborhood pixels. Scan line noise, which happens frequently in the remotely sensed scanned image, can also be considered to be peak noise if it is examined through a one dimensional window perpendicular to the scan line direction.

In order to detect peak noise, each pixel is statistically tested to determine whether its gray tone belongs to the same population as the neighborhood pixels. If it is out of the range of the 95 per cent confidence interval estimated from the gray tones in the neighborhood, it is judged as peak noise and its value is replaced by an estimated value.

Since peak noise depends on the spatial distribution of the gray tones in the neighborhood, it is important to take it into consideration when we judge peak noise. If simple mean and variance of gray tones are used as local gray tone statistics, we lose information of the higher order spatial characteristics of the gray tone distribution. In order to obtain the spatially dependent gray tone statistics, we use a facet model which fits a simple polynomial surface to the gray tone intensities in a region.[3,4] Section II introduces the basic idea of a facet model and peak noise and Section III describes the peak noise and scan line noise removal algorithm. In Section IV results applied to artificial images and real images are presented.

II. PEAK NOISE AND IMAGE MODEL

Peak noise is defined as a pixel whose gray tone intensity significantly differs from those of the neighborhood pixels. In order to measure the difference between a pixel and its neighbors, we need to estimate local gray tone statistics in the neighborhood and afterward compare those statistics with the current pixel gray tone. We should note here that peak noise is judged not from the univariate marginal distribution of the gray tone intensities in the neighborhood but on their spatial distribution. Figure 1 illustrates an example of spatial distribution dependency of peak noise. In Fig. 1 (a) and (b), the central pixel has the same gray tone "5", and the neighborhood is composed of four values {1, 2, 3, 4}. However, the peakedness of gray tone intensity is different between them. It is difficult to judge that the center pixel in Fig. 1(b) is peak noise, whereas it is more likely in Fig. 1(a). This indicates that the gray tone spatial statistics are important.

A facet model assumes a simple gray tone surface within the neighborhood. Let (r_c, c_c) be a pixel in an

*Publication no. SDA 81-10 from the Spatial Data Analysis Laboratory, Virginia Polytechnic Institute and State University.

†Dr. Yasuoka is a research scientist at the National Institute for Environmental Studies in Japan, and currently a visiting scientist at the Virginia Polytechnic Institute and State University.

‡To whom correspondence should be addressed.

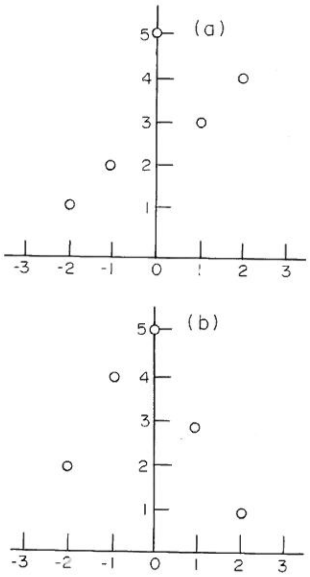

Fig. 1. Spatial distribution dependency of peak noise.

image and N be a set of neighborhood pixels, which does not contain the center pixel (r_c, c_c). Note that the use of the deleted neighborhood makes this facet approach different from that used in Haralick[3] or Haralick and Watson.[4] Let n be the number of pixels in the neighborhood N. By choosing this neighborhood, we may estimate the difference between the observed value of the center pixel's, (r_c, c_c), gray tone intensity and the value estimated from the neighboring pixels. According to the facet model, the gray tones in N are fitted to a polynomial function. A degree-one polynomial would be appropriate to illustrate the idea. The "slope facet model" is described by a linear equation

$$f(r, c) = ur + vc + w + q(r, c) \qquad (1)$$

where r is the row variable, c is the column variable, u, v and w are the slope plane coefficients and $q(r, c)$ is random stationary noise. We assume that

$$E[q(r, c)] = 0, \qquad (2)$$
$$V[q(r, c)] = s^2.$$

The least square procedure determines an estimate of u, v and w called $\hat{u}$, $\hat{v}$ and $\hat{w}$, respectively by minimizing

$$S_e = \sum_{(r,c)\in N} [\hat{u}r + \hat{v}c + \hat{w} - f(r, c)]^2. \qquad (3)$$

Without loss of generality we choose the center pixel as the origin of the relative coordinate system for each neighborhood N. Also, for simplicity, we choose the symmetric rectangular region as the neighborhood. This means

$$\sum_{(r,c)\in N} r = \sum_{(r,c)\in N} c = 0.$$

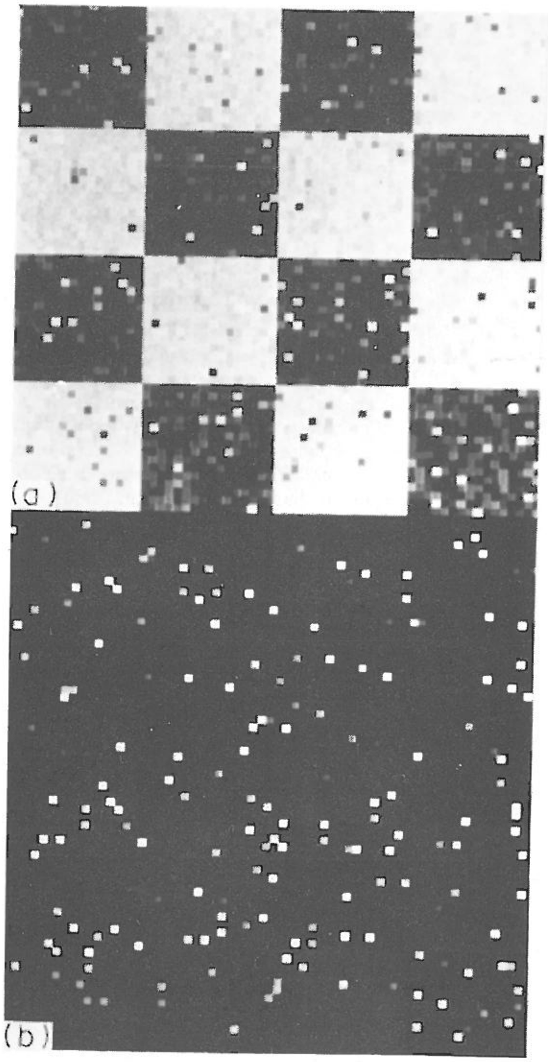

Fig. 2. A 64 × 64 test image with Gaussian noise and peak noise. (a) Test image. (b) Peak noise.

From now on, summation in equations indicates summation in the neighborhood. Under this assumption, $\hat{u}$, $\hat{v}$, $\hat{w}$ and residual error S_e are given by

$$\hat{u} = \sum f(r, c)\, r / \sum r^2 = u + \sum q(r, c)\, r / \sum r^2, \qquad (4)$$

$$\hat{v} = \sum f(r, c)\, c / \sum c^2 = v + \sum q(r, c)\, c / \sum c^2, \qquad (5)$$

$$\hat{w} = \sum f(r, c)/n \quad = w + \sum q(r, c)/n \qquad (6)$$

$$S_e = \sum [\hat{f}(r, c) - f(r, c)]^2,$$
$$= \sum q^2(r, c) - (\hat{u} - u)^2 + \sum r^2 - (\hat{v} - v)^2$$
$$+ \sum c^2 - (\hat{w} - w)^2 n \qquad (7)$$

where $\hat{f}(r, c)$ is the estimate of the gray tone at pixel (r, c) by

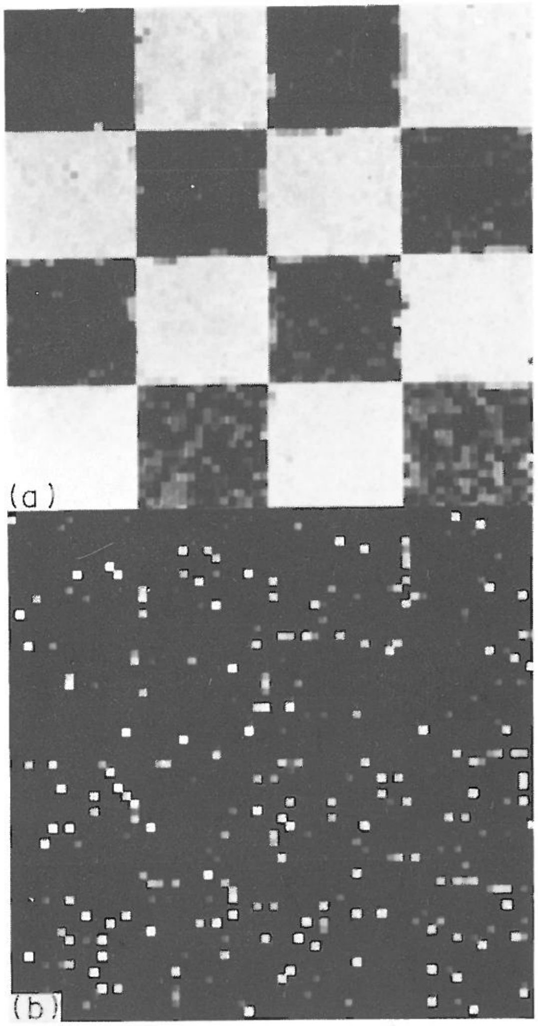

Fig. 3. The result of peak noise removal. (a) Output after two iterations of peak noise removal with a 3 × 3 window. (b) Absolute value of the differences between the original and the noise removed image.

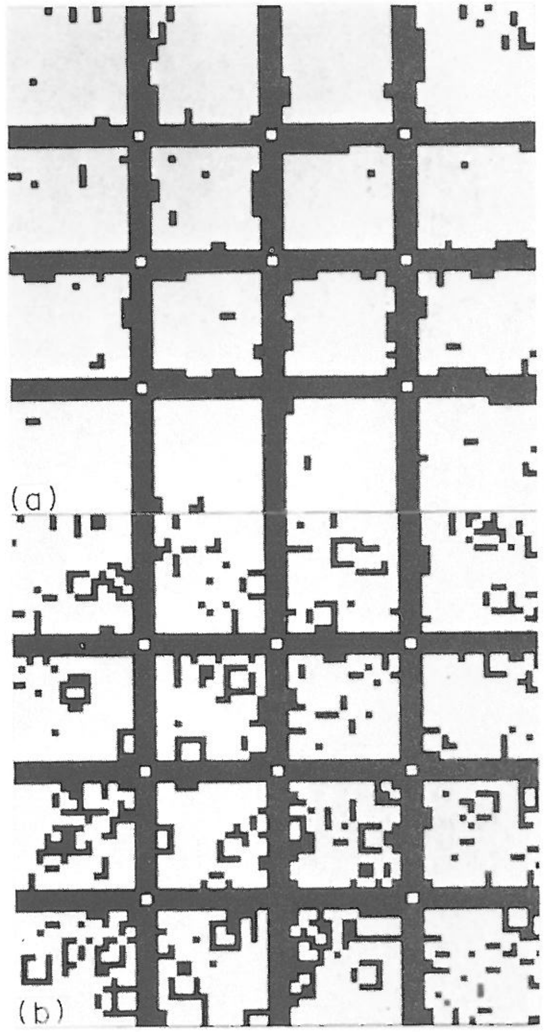

Fig. 4. Comparison of the output images after smoothing, enhancing and thresholding for (a) the peak noise removed image, (b) the original image.

$$\hat{f}(r, c) = \hat{u}r + \hat{v}c + w. \quad (8)$$

Note that $\hat{u}$, $\hat{v}$, $\hat{w}$ and S_e reflect the linear spatial distribution of gray tones in the neighborhood.

III. PEAK NOISE REMOVAL

1. *Estimation of difference*

The difference between the true gray tone and the estimated gray tone at the center pixel is given by

$$e(o, o) = \hat{w} - w - q(0, 0)$$
$$= \sum q(r, c)/n - q(o, o) \quad (9)$$

where n is the number of pixels in the neighborhood. Suppose that the center pixel is not peak noise, i.e. it belongs to the same population as the rest of the neighborhood. Then, from equation (7), $e(o, o)$ is normal because $q(r, c)$ is normal in N and at (o, o). Using the fact that

$$E[(\hat{w} - w)] = 0,$$
$$E[(w - w)^2] = s^2/n,$$

we may obtain

$$E[e(o,o)] = E[(\hat{w} - w) - q(o,o)] = 0,$$
$$V[e(o, o)] = E[\{(\hat{w} - w) - q(o, o)\}^2]$$
$$= E[(\hat{w} - w)^2] - 2E[q(o, o)(\hat{w} - w) + E[q(o, o)^2]$$
$$= (1 + 1/n)s^2,$$

since $E[q(o, o)(\hat{w} - w)] = 0$ because $q(o, o)$ is not included in the calculation of s^2 and is independent of noise in the neighborhood N. It follows that if the

Fig. 5. A 235 × 235 real image with peak noise.

Fig. 6. The result of peak noise removal of the image in Fig. 5 (after two iterations with a 3 × 3 window).

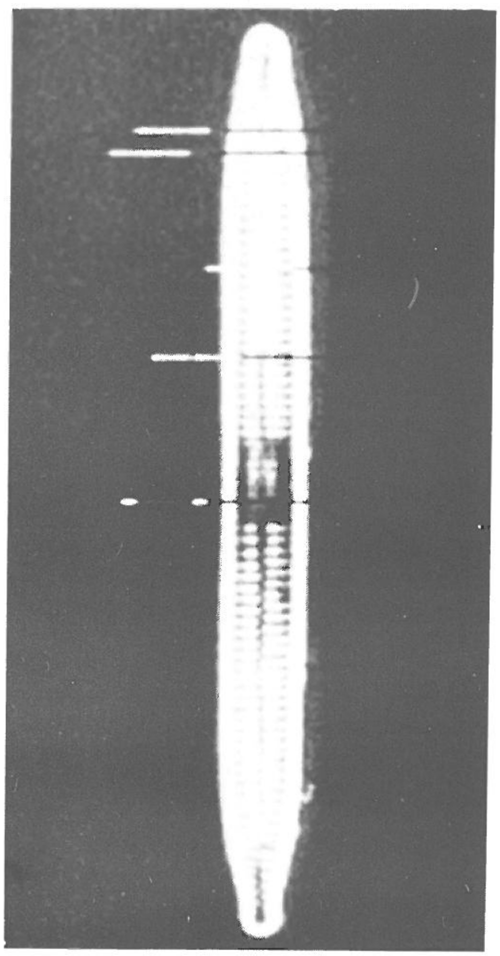

Fig. 8. A 256 × 128 diatom image with scan line noise.

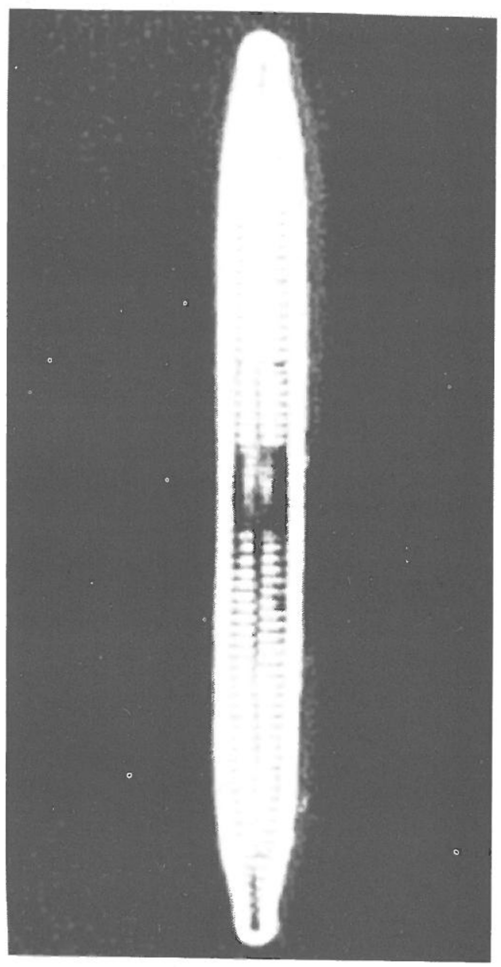

Fig. 9. The result of scan line noise removal of the image in Fig. 8 with a 7 × 1 window.

Fig. 7. The results of median filtering of the image in Fig.

center pixel belongs to the same population as the neighborhood, $e(o, o)$ is normally distributed with zero mean and $(1 + 1/n)s^2$ variance, i.e.,

$$u = \frac{\hat{f}(o, o) - f(0, 0)}{\sqrt{(1 + 1/n)s^2}} \quad (10)$$

follows normal distribution $N(0; 1)$.

Since s^2 is the unknown noise variance, its estimate, S_e, is estimated from the neighborhood pixels. From equation (7), we easily obtain

$$E[S_e] = ns^2 - 3s^2 = (n - 3)s^2 \quad (11)$$

This means that $V_e = S_e/(n - 3)$ is an unbiased estimate of s^2. Therefore, by replacing s^2 in equation (10) by V_e, we find that

$$t = \frac{\hat{f}(o, o) - f(o, o)}{\sqrt{(1 + 1/n)V_e}} \quad (12)$$

follows the t-distribution with $(n - 3)$ degrees of freedom.

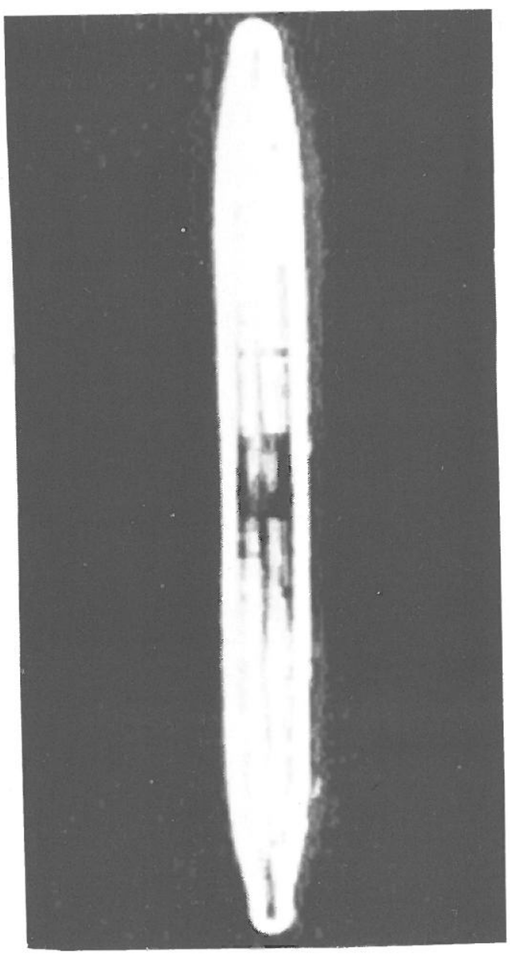

Fig. 10. The result of median filtering of the image in Fig. 8 with a 3 × 1 window.

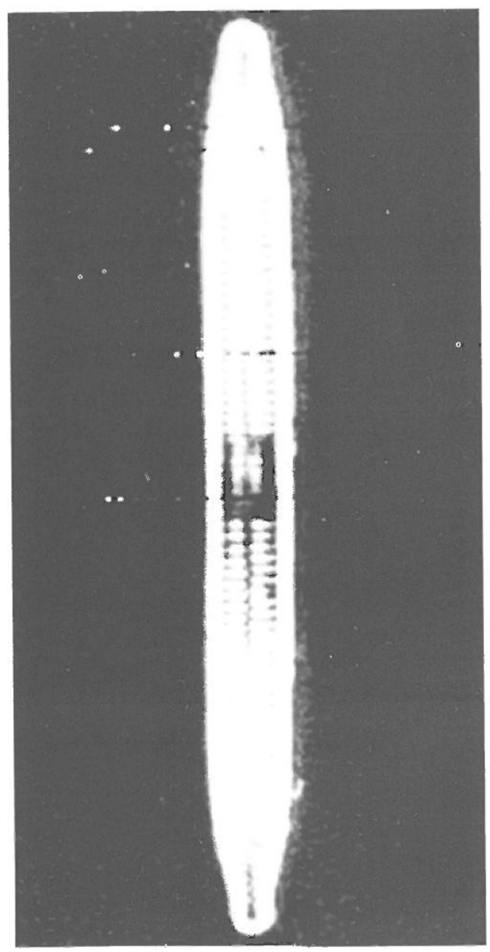

Fig. 11. The result of scan line noise removal based on a simple thresholding operation.

(2) *Test of hypothesis* $\hat{f}(o, o) = f(o, o)$

Previous discussion leads to the fact that peak noise can be detected by testing the hypothesis

$$H_o: \hat{f}(o, o) = f(o, o) \quad (13)$$

If H_o is accepted, i.e. the observed gray tone intensity $f(o, o)$ most likely belongs to the same population as the neighborhood pixels, then $f(o, o)$ is declared not to be peak noise. If H_o is rejected, then $f(o, o)$ is most likely peak noise. Hypothesis H_o is tested by using the t-statistic of equation (12). If we take 95 percentiles, the test is given by

$$H_o \text{ is accepted}: t < t(n-3, 0.05)$$
$$H_o \text{ is rejected}: t > t(n-3, 0.05) \quad (14)$$

where $t(n-3, 0.05)$ is 95 percentiles of the t-distribution with $(n-3)$ degrees of freedom. Notice that the 95 per cent confidence interval of the center gray tone is given by

$$\hat{f}(o, o) - \sqrt{(1+1/n)V_e}\, t(n-3; 0.05) < f(o, o)$$
$$< \hat{f}(o, o) + \sqrt{(1+1/n)V_e}\, t(n-3; 0.05). \quad (15)$$

If H_o is rejected, $f(o, o)$ is judged as peak noise and replaced by the estimated value $\hat{f}(o, o)$.

As a special case, if we choose a one-dimensional neighborhood window we may get a test of hypothesis for scan line noise. In the one-dimensional case, the slope facet model is reduced to

$$f(r) = wr + v + q(r). \quad (16)$$

Utilizing the fact that $V_e = S_e/(n-2)$ is an unbiased estimate of s^2, we find that

$$t = \frac{\hat{f}(o) - f(o)}{\sqrt{(1+1/n)V_e}} \quad (17)$$

follows the t-distribution with $(n-2)$ degrees of freedom. By testing the hypothesis

$$H_o: \hat{f}(o) = f(o)$$

we can detect scan line noise.

For example, consider the one-dimensional case in Fig. 1 (a) and (b), where $N = (-2, -1, 1, 2)$, $n = 4$ and $f(o) = 5$. In Fig. 1(a), $f(-2) = 1$, $f(-1) = 2$, $f(1) = 3$ and $f(2) = 4$, therefore, we obtain

$$\hat{f}(r) = \hat{w}r + \hat{v}$$
$$= (\sum f(r)r / \sum r^2)r + (\sum f(r)/n) = 0.7r + 2.5$$

and

$$S_e = \sum [\hat{f}(r) - f(r)]^2 = 0.1$$

from equation (16), and

$$t = \frac{12.5 - 5.01}{\sqrt{(1+1/4) \times 0.1/2}} = 10.0 > t(2:0.05) = 2.92.$$

Therefore, the hypothesis H_o is rejected and $f(o)$ is judged as peak noise (its value is replaced by $f(o) = 2.5$).

Similarly, in Fig. 1(b) we get

$$\hat{f}(r) = -0.3r + 2.5$$

and

$$S_e = 4.10.$$

Since

$$t = 1.56 < t(2:0.05) = 2.92,$$

the hypothesis H_o is accepted and $f(o)$ is considered to be in the same population of the neighborhood. This result corresponds with the intuitive interpretation of peak noise mentioned in Section II.

IV. RESULTS

Figure 2(a) shows a test image with Gaussian noise and peak noise. The image is a 64 × 64 checker board with 16 × 16 checks and each gray tone intensity is 100 or 200. Peak noise randomly chosen from a uniform distribution, is added to approximately 200 randomly chosen pixels (5 per cent of the whole image) [see Fig. 2(b)]. The mean standard deviation of the Guassian noise is set to 0 and 20, respectively. Figure 3(a) illustrates an output after two iterations of peak noise removal with a 3 × 3 window based on the slope facet model. Figure 3(b) illustrates the difference between the images in Fig. 2(a) and Fig. 3(a). This shows the replaced pixels.

In Fig. 3(b), besides peak noise, some pixels which are not peak noise are detected as peak noise (the type I error). This is because we use the 95 percentiles, and if we use higher percentiles, say 99, this type I error will decrease. However, in this case, the type II error (pixels which are peak noise but which are not detected as peak noise) will increase. The object of this method is to remove peak noise, so we must avoid the type II error.

In order to clarify the noise removal effect, in Fig. 4(a) we show an output after the smoothing, enhancing and thresholding of the peak noise removed image in Fig. 3(a). Figure 4(b) shows an output after the smoothing, enhancing and thresholding of the original image without noise removal. These two outputs show that peak noise is removed effectively. Here we use 3 × 3 box averaging for smoothing the image and Roberts gradient for edge enhancing the image.

Figure 5 shows a 235 × 235 image with peak noise. Peak noise is randomly chosen from a uniform distribution for 5 percent of the pixels. The noise value for these pixels is also randomly chosen from a uniform distribution. An output after two iterations of peak noise removal with a 3 × 3 window is shown in Fig. 6. Figure 7 illustrates an output after two iterations of median filtering with a 3 × 3 window. The output from the median filter shows more image degradation, while the output from the peak noise removal algorithm keeps the fine structures in the image while successfully removing the peak noise.

Figure 8 illustrates an image of a one-celled animal called a diatom. It has scan line noise. Figure 9 shows an output after peak noise removal with a 7×1 window based on the slope facet model. The output indicates that this method is also effective for scan line noise removal. Figure 10 illustrates the output after median filtering with a 5×1 window. It shows considerable image degradation. A 7×1 window would have more degradation. Figure 11 shows an output after scan line noise removal based on a simple thresholding operation. Here, each pixel value is compared with the difference between the previous line and the following line. If the difference between them exceeds the threshold, the value is judged as scan line noise and replaced by the average of the previous and the following value. If the difference between the previous and the following value exceeds another threshold, the above comparison is not performed and the current pixel value is kept. In this scheme, it is difficult to select two absolute threshold values for the whole image and because of that difficulty, the output has some detection errors while the fine structure of the image is preserved fairly well.

V. CONCLUSION

We have discussed a method of removing peak noise by utilizing the facet model with the deleted neighborhood. In this method, peak noise is statistically detected by the use of a t-test. The facet can be a flat facet, slope facet or higher order facet. We illustrated the technique using the slope facet. We showed how the technique performs better than median filtering in that median filtering has a higher image degradation.

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About the Author—YOSHIFUMI YASUOKA received his Ph.D degree from the University of Tokyo in 1974. Since 1974 he has been working in the Division of Environmental Studies, Japan, and has also lectured at the University of Tskuba in the Information Sciences Department. He has published papers in the area of the computer processing of remotely sensed data and is interested in the development of image processing software systems.

About the Author—ROBERT M. HARALICK was born in Brooklyn, New York, on 30 September 1943. He received the BS degree from the University of Kansas in 1966. He has worked with Autonetics and IBM. In 1965 he worked for the Center for Research, University of Kansas, as a research engineer and in 1969, when he completed his PhD at the University of Kansas, he joined the faculty of the Electrical Engineering Department where he served as a Professor from 1975 to 1978. In 1979, Dr. Haralick joined the faculty of the Electrical Engineering Department at Virginia Polytechnic Institute and State University, where he is now a Professor and director of the Spatial Data Analysis Laboratory. Dr. Haralick has carried out research in pattern recognition, multi-image processing, remote sensing, texture analysis, data compression, clustering, artificial intelligence and general systems theory. He is responsbile for the development of GIPSY (General Image Processing System), a multi-image processing package which runs on a minicomputer system.

He is a member of the Institute of Electrical and Electronic Engineers, the Association for Computer Machinery, Sigma Xi, the Pattern Recognition Society and the Society for General Systems Research.