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TEXTURE PATTERN IMAGE GENERATION BY REGULAR MARKOV CHAIN

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Abstract – A method of graytone texture pattern generation using a regular Markov chain is presented. The procedure arranges the generated gray tones in a sequence along a scan line. The transition probability matrix of the Markov chain directly determines the spatial co-occurrence probabilities of gray tones in the generated image. The generated image can be rotated and arithmetically combined to produce images of additional texture patterns. The paper illustrates the variety of texture which can be produced by this method.

Texture   Image generation   Markov chain   Image synthesis   Probability transformation

1. INTRODUCTION

The generation of texture in computer synthesized images is important to graphics.[1] It is also important to image processing people who do texture feature extraction on remotely sensed imagery as well as medical imagery. Comparative studies of the efficiency and accuracy of those techniques have been done to identify their advantages.[2,3] In such studies, samples of textures are usually taken from real image data. This is reasonable because the ultimate purpose is to analyze real data. But, on the other hand, conclusions of such studies might sometimes be one-sided or not generalizable because of the limited number and variety of the data samples.

To make a thorough comparison of each technique, we can supplement the real data by synthesized texture patterns having specified statistics. Then, by using the synthesized texture patterns, the characteristics of each technique can be investigated in the usual manner to provide stronger conclusions about the advantages of each textural feature extraction method.

There are, however, only a few studies about texture pattern analysis and/or synthesis.[4–7] These studies use either a Markov chain approach or an autoregression approach.

In this paper we generate texture using Markov chains. A texture image can be synthesized using the outcomes of gray tone values from a regular Markov chain by arranging these outcomes in a sequential manner along an image line. The major advantages of this method are a simple synthesis process and direct specification of spatial co-occurrence matrix of synthesized texture. The generation procedure can be extended by selecting according to some rule or pattern the outcomes from a set of simultaneously operating Markov processes, and then applying a probability transformation. Synthesized textures by this kind of extension can have more generalized structures and variety in their patterns.

The next section describes the mathematical notation used in the paper, and theory of regular Markov chains is briefly summarized. In Section 3, the method of texture synthesis by a simple regular Markov chain is described. The extended methods are discussed in section 4, and examples of synthesized textures are given in section 5.

2. NOTATION AND REGULAR MARKOV CHAIN

Pictorial information of an image is represented as a function of row-column variables (r, c). The image in its digital form is usually stored in the computer as a two-dimensional array. If $L_r = \{1, 2, \ldots, N_r\}$ and $L_c = \{1, 2, \ldots, N_c\}$ are the row and column spatial domains, then $L_r \times L_c$ is the set of resolution cells and the digital image is a function I which assigned some gray tone value $G = \{1, 2, \ldots, N\}$ to each and every resolution cell; $I: L_R \times L_C \to G$.

The frequency $p(i)$ of a grey tone i occurring in the image is defined by

$$p(i) = \frac{\#[(k,h) \varepsilon L_r \times L_c | I(k,h) = i]}{N_r \cdot N_c}.$$

Using these frequency values, we define an occurrence probability vector $\tilde{P}$ and an occurrence probability matrix $\hat{P}$ as follows:

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$$\tilde{P} = \begin{bmatrix} p(1) \\ p(2) \\ \vdots \\ p(N) \end{bmatrix}$$

$$\hat{P} = \begin{bmatrix} p(1) & & & \\ & p(2) & [0] & \\ & [0] & \ddots & \\ & & & p(N) \end{bmatrix}. \quad (1)$$

Spatial co-occurrence probability $p(i,j;\, d, \delta)$ is defined by using parameters of a distance d and a direction δ. For a case of $\delta = 0$, it can be described as

$$p(i,j;d,0) = \#\{[(k,h),(m,n)]\,\varepsilon(L_r \times L_c) \times (L_r \times L_c) | m-k=0, n-k=d, I(k,h)=i, I(m,n)=j\}/(N_r \cdot N_c).$$

There are eight directions of δ. Each value of δ is an angle of clockwise rotation from the positive horizontal direction and their co-occurrence probabilities are similarly described by following the above. A more detailed discussion about the concept can be found in Haralick *et al.*[8]

From the definition it is easily justified:

$$p(i,j;d,\delta) = p(j,i;d,\delta+180), \quad \delta = 0,45,90,135. \quad (2)$$

A spatial co-occurrence probability matrix $P(d,\delta)$ has spatial co-occurrence probabilities $P(i,j;\,d,\delta)$ as its entries:

$$P(d,\delta) = \begin{bmatrix} p(1,1;d,\delta), & p(1,2;d,\delta) & \dots & p(1,N;d,\delta) \\ p(2,1;d,\delta), & p(2,2;d,\delta) & \dots & p(2,N;d,\delta) \\ \vdots & \vdots & & \vdots \\ p(N,1;d,\delta), & p(N,2;d,\delta) & \dots & p(N,N;d,\delta) \end{bmatrix} \quad (3)$$

$$(\delta = 0,45,90,\dots,315).$$

From (2) and (3), we must have

$$P(d,\delta+180) = P^T(d,\delta), \quad \delta = 0,45,90,135. \quad (4)$$

Let us define a transition probability matrix $M(d,\delta)$ having for each entry $m(i,j;d,\delta)$ the conditional probability of a gray tone transition from gray tone i to gray tone j between points which are d apart in the the direction of δ. Then

$$M(d,\delta) = \{m(i,j;d,\delta)\} = \hat{P}^{-1}P(d,\delta)$$

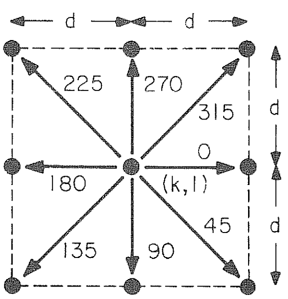

Fig. 1. Convention for the direction and distance relationships for a spatial co-occurrence probability matrix.

and

$$m(i,j;d,\delta) = \frac{p(i,j;d,\delta)}{p(i)}, \quad (5)$$

$$\delta = 0,45,90,\dots,315.$$

Then from (4) and (5), it becomes

$$M(d,\delta+180) = \hat{P}^{-1}P^T(d,\delta) \quad (6)$$

Let M be a transition matrix of a finite Markov chain. The matrix M is regular if and only if for some integer L, M^L has no zero entries. A finite Markov chain having a regular transition matrix is called a regular Markov chain.

A regular Markov chain is one in which any of its states can exist after some number of steps L, no matter what the starting state is. In other words, a regular Markov chain has no transient sets, and has a single ergodic set with only one cycle.

The next two theorems play very significant roles in calculating spatial co-occurrence probability matrices of generated textures in the next section. Their proofs can be found in.[9]

Theorem 1. If M is a $N \times N$ regular transition matrix, then there exists a matrix A satisfying

(i) $\lim_{L\to\infty} M^L = A$,

(ii) each row of A is composed of the same probability vector of

$$\alpha = [\alpha_1, \alpha_2, \dots, \alpha_N],\ \alpha_n \geqslant 0 \text{ and } \sum_{n=1}^{N} \alpha_n = 1$$

that is

$$A = \xi\alpha \text{ where } \xi = (1,1,\dots,1).$$

The matrix A is called a limiting matrix of M.

Theorem 2. If M is a regular transition matrix and A and α are as given in Theorem 1, then

(i) for any probability vector Π,

$$\lim_{L\to\infty} \pi M^L = \alpha,$$

(ii) the vector α is the unique probability vector such that

$$\alpha M = \alpha, \text{ and}$$

(iii) $MA = AM = A$

3. IMAGE GENERATION BY A SINGLE REGULAR MARKOV CHAIN AND ITS STATISTICS

Now consider a regular Markov chain such that it has the outcomes of gray tone values $G = (1,2,\dots,N)$ and a transition matrix M of

$$M = \begin{bmatrix} m(1,1) & m(1,2) & \dots & m(1,N) \\ m(2,1) & m(2,2) & \dots & m(2,N) \\ \vdots & \vdots & & \vdots \\ m(N,2) & m(N,2) & \dots & m(N,NO) \end{bmatrix}$$

Each $m(i,j)$ is the conditional probability of generating a gray tone value j when the previous gray tone is i.

We will assume that the spatial domain of the synthesized image is large enough so that

$$M^{N_c} \cong A \quad \text{and} \quad M^{N_r} \cong A \tag{7}$$

Then the matrix $\hat{P}$ defined in (1) is given by:

$$\hat{P} = \begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & [0] \\ & & \ddots & \\ [0] & & & \ddots \\ & & & & \alpha_N \end{bmatrix}$$

The texture synthesis procedure is described as follows.

Step 1. Using an arbitrary initial gray tone value g^o to initialize the regular Markov chain. Let $g(k; M, g^o)$ be the kth outcome from the Markov chain.

Step 2. Arrange the outcomes to generate an image by

$$I(k,l) = g(N_c(k-1) + (l-1);\ M, g^o)$$

with

$$I(1,1) = g(0, M; g^o) = g^o \tag{8}$$

In the above, outcome gray tone values are arranged sequentially across a horizontal line as shown in Fig. 2a.

As the scanning direction is 0-degree, the transition probability matrix in the 0-degree direction distance d is simply given by

$$M(d,0) = M^d \tag{9}$$

So from (5) and (9), its spatial co-occurrence probability matrix in the 0-degree direction is given

$$P(d,0) = \hat{P}M^d \tag{10}$$

The spatial co-occurrence probability matrix for the 90-degree direction is determined using gray tone values between points of a same column but a different row, i.e. (k,l) and $(k+d,l)$. From the generation, the transition probability of gray tones between points drawn apart is given by M^{dN_c}, so

$$M(d,90) \cong M^{dN_c} = A \tag{11}$$

Thus the spatial co-occurrence probability matrix in the 90-degree direction is

$$P(d,90) = \hat{P}A = \alpha^T\alpha$$

so that

$$P(i,j;d,\delta) = \alpha_i\alpha_j \tag{12}$$

By using similar reasoning, transition probability matrices of 45-degree and the 135-degree direction are given by

$$M(d,45) = M^{d(N_c+1)} \cong A,$$

and

$$M(d,135) = M^{d(N_c-1)} \cong A$$

respectively. Accordingly we have

$$\begin{cases} P(d,45) = \hat{P}A = \alpha^T\alpha, & \text{and} \\ P(d,135) = \hat{P}A = \alpha^T\alpha \end{cases} \tag{13}$$

From (6),

$$P(d,180) = P^T(d,0) = (M^T)^d P\cdot$$
$$P(d,225) = P^T(d,45) = \alpha^T\alpha,$$
$$P(d,270) = P^T(d,90) = \alpha^T\alpha,$$
$$P(d,315) = P^T(d,135) = \alpha^T\alpha.$$

Of course, the generation does not have to proceed horizontally. When different scanning directions, as shown in Fig. 2 are used to generate an image, the spatial co-occurrence probability matrices are generalized as follows.

$$P(d,\delta) = \begin{cases} \hat{P}M^d; & \text{when } \delta \text{ is in the scanning direction,} \\ (M^T)^d P; & \text{when } \delta \text{ is in the anti-scanning direction, and} \\ \alpha^T\alpha; & \text{other cases} \end{cases}$$

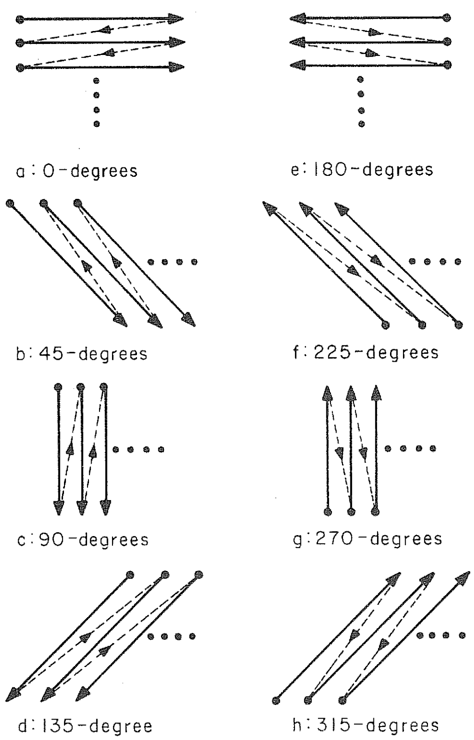

Fig. 2. Scanning directions of gray tone outcomes arrangement.

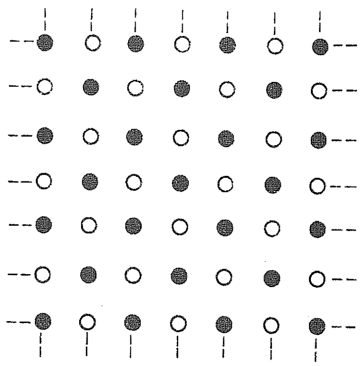

Fig. 3. Outcomes arrangement in a weaving model. Note N_r and N_c are odd.

Thus texture patterns with various spatial co-occurrence probability matrices are realizable by chosing appropriate M matrices.

4. EXTENDED SYNTHESIS METHODS

The generation method stated in the previous section can be extended by using multiple Markov chains and a rule or pattern for selections among them. Four different methods are described here. Generalizing the structure of statistics of synthesized images, these generation methods can produce textures of a greater variety.

1. *An image by a weaving method*

In a weaving method, an image is synthesized by multiple regular Markov processes whose outcomes are arranged alternately in lines of an image as in Section 3.

Let us consider two independent regular Markov chains, say chain No. 1 and chain No. 2. Let quantities of M, α, A, $\hat{P}$ defined in Section 3 have subscripts denoting which Markov process they describe.

One of the simplest methods for arrangement is a plain weaving of Fig. 3, in which the solid circles and the empty circle represent the outcomes from chain No. 1 and chain No. 2, respectively. The scanning direction is assumed to be 0-degree and assume

$$M^{(N_c-1)/2} \simeq A \qquad \text{and} \qquad M^{(N_r-1)/2} \simeq A$$

in place of (7). Now let us consider the spatial co-occurrence probability matrix of the image.

From the assumptions, chain No. 1 and chain No. 2 are independent of each other and have occurrence probability vectors of gray tone values as $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ respectively.

For $\delta = 0$ and $d = 1$, the co-occurrence probability matrix from chain No. 1 outcomes to chain No. 2 outcomes is simply given by $\alpha_1^T\alpha_2$, and the one from chain No. 2 outcomes to chain No. 1 outcomes is $\alpha_2^T\alpha_1$. Thus its whole co-occurrence probability matrix is the average of these and is given by:

$$P(1,0) = \tfrac{1}{2}(\alpha_1^T\alpha_2 + \alpha_2^T\alpha_1)$$

The consideration is applicable to the co-occurrence probability matrices between the outcomes of two chains. These cases exist in the 0 and 90 degree direction with distances of odd numbers. So we have

$$P(2n+1,0) = P(2+1,90) = \tfrac{1}{2}(\alpha_1^T\alpha_2 + \alpha_2^T\alpha_1)$$

$$n = 0, 1, 2, \ldots, \frac{N_c - 1}{2}.$$

When the distance has an even value in the 0-degree direction, co-occurrence probabilities are defined between the outcomes from the same chain. Let the distance be $2l$, then from (10), the spatial co-occurrence probability matrices for the outcomes of chain No. 1 and chain No. 2 are given by P_1M_1 and P_2M_2. Thus the whole probability matrix is

$$P(2l,0) = \tfrac{1}{2}\{P_1M_1 + P_2M_2\}.$$

$$l = 1, 2, \ldots, \frac{N_c - 1}{2}.$$

In the directions of 45 and 135 degrees, the outcomes of each chain are arranged in parallel. Referring the co-occurrence probabilities to (13), we have

$$P(d,45) = P(d,135) = \tfrac{1}{2}(\alpha_1^T\alpha_1 + \alpha_2^T\alpha_2). \qquad (14)$$

The last is the one in 90-degree direction with even distances. In this case, the spatial co-occurrence probability matrix is defined between the outcomes from the same chains after a large enough distance of N_c. Thus as in the case of (14), we have

$$P(2,90) = \tfrac{1}{2}(\alpha_1^T\alpha_1 + \alpha_2^T\alpha_2).$$

The spatial co-occurrence probability matrix are summarized as follows:

$$P(d,0) = \begin{cases} \tfrac{1}{2}(\alpha_1^T\alpha_2 + \alpha_2^T\alpha_1) & \text{for odd } d \\ \tfrac{1}{2}(\hat{P}_1M_1^{d/2} + \hat{P}_2M_2^{d/2}) & \text{for even } d \end{cases}$$

$$P(d,45) = \tfrac{1}{2}(\alpha_1^T\alpha_1 + \alpha_2^T\alpha_s). \qquad (15)$$

$$P(d,90) = \begin{cases} \tfrac{1}{2}(\alpha_1^T\alpha_2 + \alpha_2^T\alpha_1) & \text{for odd } d \\ \tfrac{1}{2}(\alpha_1^T\alpha_1 + \alpha_2^T\alpha_2) & \text{for even } d \end{cases}$$

$$P(d,135) = \tfrac{1}{2}(\alpha_1^T\alpha_1 + \alpha_2^T\alpha_2).$$

The cases of 180, 225, 270, and 315 degrees are found by (6). In the previous result, ranks of spatial co-occurrence probability matrix are reduced to one if it is not in the scanning direction, but those in (15) can have larger ranks.

Although only one case is analyzed in the above, the synthesis procedure can be generalized by different scanning directions, different arrangement rules, or using many Markov chains, etc. The outcome sequences from each Markov chain can be compared to strings for weaving textures in this method.

2. *An image by composite method*

In a composite synthesis method, we use a regular Markov chain and a set of random gray tone value generators. Conveniently giving index numbers to the generators, let us describe a probability vector of their outcomes to be:

$$\beta_i = [\beta_{i1}, \beta_{i2}, \ldots, \beta_{iN}], \qquad i = 1, 2, \ldots, M$$

where β_{ij} means an occurrence probability of gray tone value j from the i-th random gray tone value generator. The regular Markov chain has the set of outcomes of $\{1, 2, \ldots, M\}$, which are the index symbols of the random generators.

In the first stage of the texture synthesis, the regular Markov chain generates an image by the same method as stated in Section 3 such that each resolution cell has one of the index symbols of the random generators. We call such an image a symbolic image. Describe the resulting spatial co-occurrence probability matrix of symbols as

$$S(d,\delta) = \begin{bmatrix} S(1,1;d,\delta) & S(1,2;d,\delta) & \ldots & S(1,M;d,\delta) \\ S(2,1;d,\delta) & S(2,2;d,\delta) & \ldots & S(2,M;d,\delta) \\ \vdots & \vdots & & \vdots \\ S(M,1;d,\delta) & S(M,2;d,\delta) & \ldots & S(M,M;d,\delta \end{bmatrix}. \quad (16)$$

which is already calculated in Section 3.

Then, in the second stage, a texture image is synthesized. A grey tone value of each resolution cell is to be supplied by the random gray tone value generator having the same index symbol in the symbolic image generated in the first stage. That is, the regular Markov chain indirectly participates in the gray tone values by specifying their suppliers.

Now let us consider the statistics of the synthesized textures. Given a transition from an outcome of the i-th gray tone value generator to an outcome of the j-th generator, the conditional transition probability matrix is:

$\beta_i^T \beta_j$—this is a conditional probability matrix.

From the generation of the symbolic image, such a transition can occur with a probability of $S(i,j;d,\delta)$. Thus, in the spatial co-occurrence probability matrix of the synthesized image, this particular transition contributes an amount of

$$S(i,j;d,\delta)\beta_i^T \beta_j$$

Accordingly, the spatial co-occurrence probability of the texture must be the total of all those combinations, i.e.

$$P(d,\delta) = \sum_i^M \sum_j^M S(i,j;d,\delta)\beta_i^T \beta_j. \quad (17)$$

Looking from this composite synthesis method, we can identify the synthesis method of Section 3 to be its special case. That is, if we use N gray tone generators with occurrence probabilities such that

$$\beta_i = [0, \ldots, 0, \underset{\substack{\uparrow \\ i\text{-th component}}}{1}, 0, \ldots, 0], \qquad i = 1, 2, \ldots, N. \quad (18)$$

then $P(d,\delta)$ of (17) is reduced to S of (16). The gray tone generator with β_i of (18) is not a random one any more, but always supplies gray tone value i.

3. *Compound of images*

Let us assume that I_1 and I_2 are images generated by regular Markov chains No. 1 and No. 2 respectively. The a compound image I is defined as

$$I(r,c) = I_1(r,c) + I_2(r,c), \quad \text{for all } (r,c)\varepsilon L_r \times L_c.$$

Since the gray tone values in I_1 and I_2 vary from 1 to N, those in I vary from 2 to $2N$.

Now consider the spatial co-occurrence probabilities in the compound image. From the generation procedure, a co-occurrence transition from a gray tone value m to n in I must be provided by co-occurrence transitions from i to j in I_1 and from k to 1 in I_2 such that

$$\begin{cases} i + k = m \\ j + 1 = n \end{cases}.$$

Thus its co-occurrence probability is given by

$$p(m,n;d,\delta) = \sum_{w=\max\{1,n-N\}}^{\min(N,n-1)} \sum_{t=\max\{1,m-N\}}^{\min(N,m-1)} p_1(t,w;d,\delta).$$
$$p_2(m-t,n-w;d,\delta). \quad (19)$$

where p_1 and p_2 are spatial co-occurrence probabilities in I_1 and I_2 respectively. The spatial co-occurrence probability matrix cannot be described compactly by matrix or vector notations, but its entries can be simply calculated by computer according to (19).

By using various original images of different statistics, the compound images can have a large variety. The compound process is not necessarily restricted to two images and may incorporate many images.

4. *Probability transformation*

A probability transformation can be applied to any texture. The gray tone values of the probability transformed image are determined in a pointwise manner by the gray tone values of the original image under a specified probability of transformation. Let us describe the transformation probability matrix as

$$Q = \begin{bmatrix} q(1,1) & q(1,2) & \ldots & q(1,N) \\ q(2,1) & q(2,2) & \ldots & q(2,N) \\ \vdots & \vdots & & \vdots \\ q(N,1) & q(N,2) & \ldots & q(N,N) \end{bmatrix}. \quad (20)$$

where each $q(i,j)$ is the conditional probability of the occurrence of a gray tone value j in the transformed image when the original image has a value i.

A co-occurrence of a transition from i to j in the original image provides a co-occurrence of transition from k to 1 in the transformed image with a probability of

$$q(i,k)p(i,j;d,\delta)q(j,1).$$

Thus the total co-occurrence probability $r(k,1;\delta,d)$ in the transformed image is given by

$$r(k,1;d,\delta) = \sum_{i,j} q(i,k)p(i,j;d,\delta)q(j,1) = Q_k^T P(d,\delta)Q_1$$

where Q_k, $k = 1,2,\ldots,N$, is the k-th column of Q and the spatial co-occurrence probability matrix of the transformed image is

$$R(d,\delta) = Q^T P(d,\delta)Q \quad (21)$$

That is, the R is a congruent matrix of P associated with Q.

When, in each row of Q, only one element is 1 and the others are 0, the transformation performs a change of gray tone values deterministically.

In the transformation, the number of gray tone values of the transformed image is not necessarily equal to N, but it can be any positive integer. In this case, Q must be generalized to be a rectangular matrix.

In the results of (15), (17) and (19), the rank of spatial co-occurrence probability matrices is considerably generalized from the results of Section 3. From (21), the probability transformation can easily modify the spatial co-occurrence probability matrix and reduce or increase the number of gray tone values in the transformed image. Using these further extended methods, we can obtain a greater variety of texture patterns.

5. *Examples*

In the following examples, regular Markov chains and random gray tone value generators are used in the synthesis of texture samples. Outcomes have 3 gray tone values of 1(white), 2(gray), and 3(black), and the scanning direction is 0-degrees unless otherwise stated.

(a) Markov chain 1; (MK-1) $M = \begin{bmatrix} \frac{3}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{3}{4} \end{bmatrix}$

with $\alpha = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$.

(b) Markov chain 2; (MK-2) $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$

with $\alpha = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$.

(c) Markov chain 3; (MK-3) $M = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

with $\alpha[\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$.

(d) Markov chain 4; (MK-4) $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$

with $\alpha = [\frac{2}{5}\ \frac{1}{5}\ \frac{2}{5}]$.

(e) Markov chain 5; (MK-5) $M = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \end{bmatrix}$

with $\alpha = [\frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}]$

and

(f) Random gray tone values generators

$\beta_1 = [0.7, 0.2, 0.1]$,

$\beta_2 = [0.2, 0.7, 0.1]$,

$\beta_3 = [0.1, 0.2, 0.7]$.

Figure 4 shows samples of texture patterns synthesized by MK-1 to 5. Textures 4.1 ~ 4.3 have different coarseness of texture in the horizontal direction, which come from different dominancy of diagonal elements in transition matrices. In texture 4.4, a transition of gray tone values $2 \to 2$ in the 0-degree direction does not occur. In texture 4.5, transition probability of gray tone values $2 \to 2$ is large, but those of values $1 \to 1$ and values $3 \to 3$ are small. Subsequently, the gray color is dominant in the synthesized image.

Figure 5 shows samples of textures synthesized by the plain weaving method.

Figure 6 shows samples of textures synthesized by the composite method. In the synthesis, β_1, β_2 and β_3 are associated with symbols 1, 2 and 3 respectively in the symbolic images generated by Markov chains.

Figure 7 shows samples of texture patterns synthesized by the compound method. In each case, the scanning directions in the generations of the first image and the second image are 0- and 90-degree respectively. Then two images are added together. Because each original image has 3 gray tone values, the images of the compound method have 5 gray tone values.

Figure 8 shows samples of texture patterns synthesized by the probability transformations. To texture 4.1, three kinds of probability transformations are applied. Their transition matrices are as follows:

$$Q_a = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.05 & 0.9 & 0.05 \\ 0.05 & 0.05 & 0.9 \end{bmatrix},$$

$$Q_b = \begin{bmatrix} 0.75 & 0.125 & 0.125 \\ 0.125 & 0.75 & 0.125 \\ 0.125 & 0.125 & 0.75 \end{bmatrix},$$

$$Q_c = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

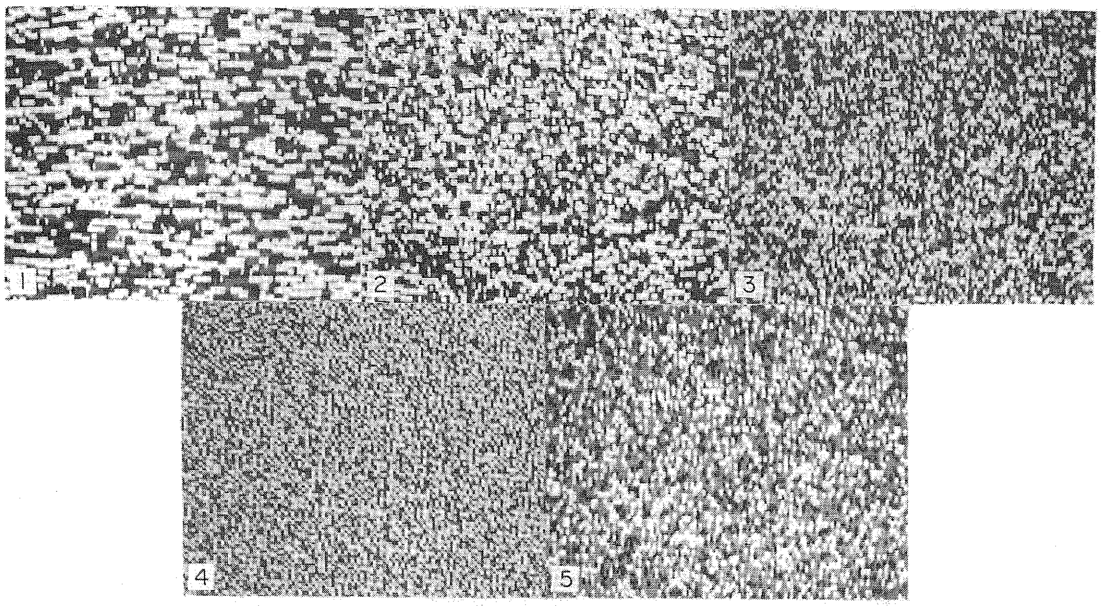

Fig. 4. Texture patterns synthesized by a single regular Markov chain.

Texture 4.1: generated by Mk-1.

Texture 4.2: generated by MK-2.

Texture 4.3: generated by MK-3.

Texture 4.4: generated by MK-4.

Texture 4.5: generated by MK-5.

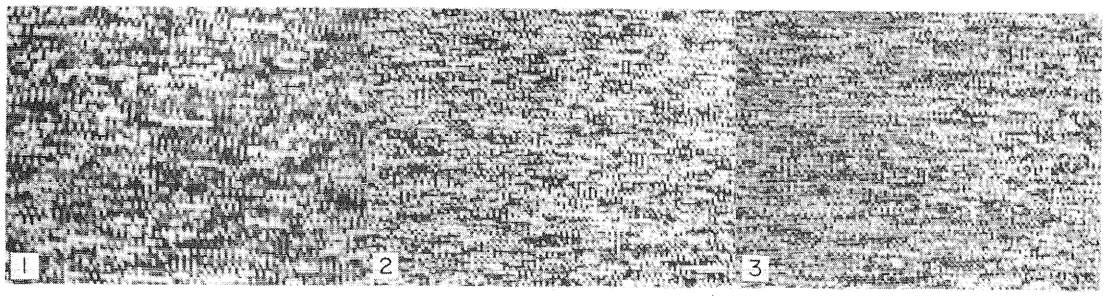

Fig. 5. Texture patterns generated by the plain weaving method.

Texture 5.1: generated by MK-1 and MK-2.

Texture 5.2: generated by MK-1 and MK-4.

Texture 5.3: generated by MK-1 and MK-5.

The textures of 8.1 has a slight distortion of patterns from Texture 4.1, because of the diagonal element dominancy in Q_a. But in the case of Texture 8.3, its original pattern can be hardly recognized.

6. *Conclusion*

We have suggested procedures for synthesizing texture images using regular Markov chains. The simplest method is to arrange outcomes of gray tone values from a Markov chain sequentially along image lines. The transition probability matrix directly determines the spatial co-occurrence probability matrices in the scanning and reverse scanning direction. Those in other directions are described using the limiting matrix.

The procedure was extended to compound images,

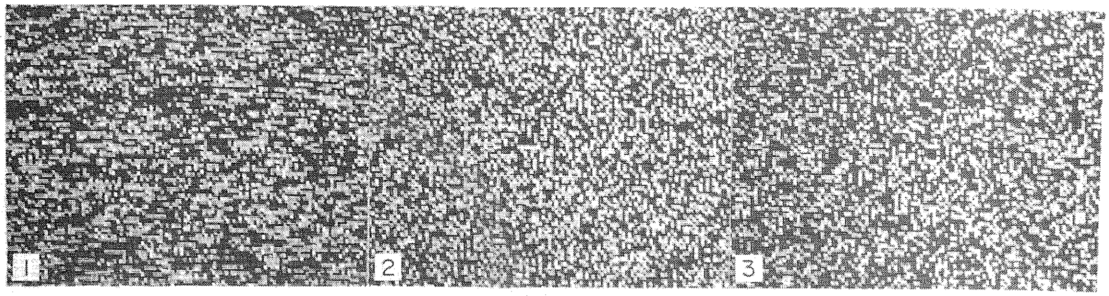

Fig. 6. Textures synthesized by the composite method with random gray tone value generators of β_1, β_2 and β_3.

Texture 6.1: the original symbolic image is generated by MK-1.

Texture 6.2: the original symbolic image is generated by MK-4.

Texture 6.3: the original symbolic image is generated by MK-5.

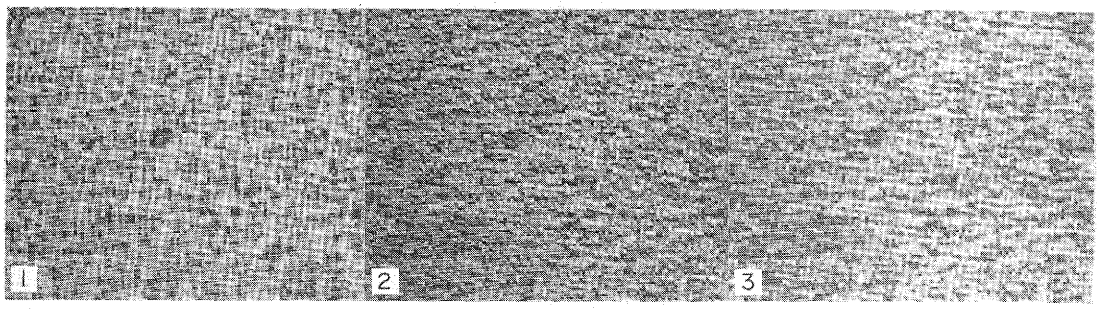

Fig. 7. Textures by the compound method.

Texture 7.1: generated by MK-1 and MK-1.

Texture 7.2: generated by MK-1 and MK-4.

Texture 7.3: generated by MK-1 and MK-5.

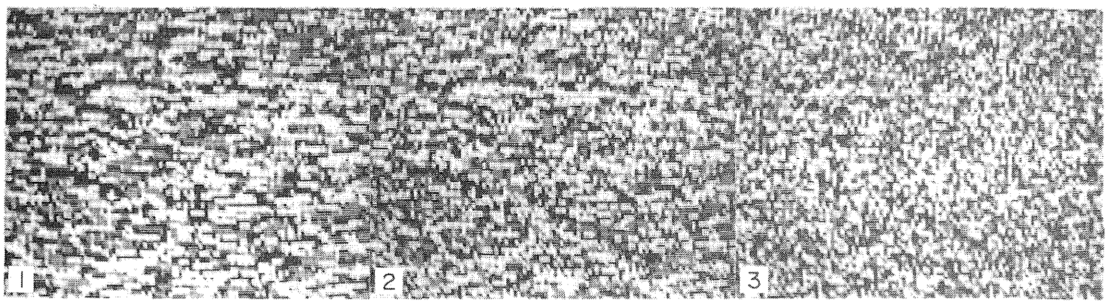

Fig. 8. Textures synthesized by the probability transformations from Texture 4.1.

Texture 8.1: generated by Q_a.

Texture 8.2: generated by Q_b.

Texture 8.3: generated by Q_c.

composite images, and those generated by multiple Markov chains. Further processing is possible to images by applying a probabilistic point operator, which can modify the statistics of images or change the number of gray tone values.

Thus by choosing appropriate transition probability matrices of Markov chains and probability transformation, texture images with various statistical values and patterns are realizable as shown in Section 4.

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