$C = 0$, $m = 0$, $N = 0$, and $\hat{N} = n_{p_1} + \cdots + n_{p_l}$. Thus, the composite statistics of the training class are

$$\hat{\mu} = \sum_{i=1}^{l} \left(\frac{n_{p_i}}{\hat{N}}\right) \mu_{p_i} \tag{9}$$

$$\hat{C} = \sum_{i=1}^{l} \left(\frac{n_{p_i} - 1}{\hat{N} - 1}\right) C_{p_i} + \sum_{i=1}^{l} \left(\frac{n_{p_i}}{\hat{N} - 1}\right) \mu_{p_i}\mu_{p_i}^T - \left(\frac{\hat{N}}{\hat{N} - 1}\right) \hat{\mu}\hat{\mu}^T. \tag{10}$$

A Computationally Simple Procedure for Imagery Data Compression by the Karhunen–Loève Method

K. SHANMUGAM AND R. M. HARALICK

Abstract—**Of the several methods that have been proposed for imagery data compression, the Karhunen–Loève procedure minimizes the mean-square error between the original and reconstructed imagery data. In spite of its optimality property, the Karhunen–Loève procedure has not been widely used because of its computational complexity. The main difficulty is in the computation of the eigenvectors and the eigenvalues of the covariance matrix of the imagery data since the dimension of the covariance matrix is usually large.**

A computationally short procedure for calculating the eigenvalues and eigenvectors of the covariance matrix is presented. We show that the eigenvalues and eigenvectors of the $N \times N$ bisymmetric covariance matrix can be obtained from the eigenvalues and eigenvectors of two $N/2 \times N/2$ submatrices. Since the eigenvector calculations are proportional to the third power of the matrix dimension, the proposed procedure reduces the computations by a factor of four.

I. Introduction

Imagery data in general contain a large amount of redundant information because of the high positive correlation between the gray levels of spatially adjacent image elements. Several imagery data compression techniques have been proposed recently for removing this redundant information [1]–[4]. Of these methods, the principal component method (based on the Karhunen–Loève expansion) minimizes the mean-square error between the original and compressed imagery data.

In the principal components method, the image is first split into a number of small mutually exclusively spatial regions or windows, and the gray levels of these regions are treated as N-dimensional vectors. (These vectors are assumed to have a mean of zero; if not, the mean vector can be calculated and subtracted from each of these vectors.) The image is then a collection of these vectors. These N-dimensional vectors $X_1,X_2,\cdots,X_k$ are then projected into some smaller r-dimensional subspace having maximal variance. In this way the N components of the original data may be expressed in terms of r components, thus achieving a data compression of N/r.

An optimal basis for the r-dimensional subspace is the set of r eigenvectors $V_1,V_2,\cdots,V_r$ corresponding to the r largest eigenvalues of the sample covariance matrix of $X_1,X_2,\cdots,X_k$. The reconstructed value of the imagery data in the jth subimage region (window) is given by

$$X_j^* = \sum_{i=1}^{r} [V_i' \cdot X_j]V_i \tag{1}$$

where V_i' denotes the transpose of V_i. The principal component procedure minimizes the mean-square error

$$E = \frac{1}{k}\sum_{i=1}^{k} \|X_i - X_i^*\|^2. \tag{2}$$

The minimum value of E for r projections is given by

$$E_r^* = \sum_{i=r+1}^{N} \lambda_i \tag{3}$$

where λ_i are the $N - r$ smallest eigenvalues of the covariance matrix.

In spite of its optimality, the principal component method has not been widely used because of its computational complexity [5]. The main difficulty is in the computation of the eigenvalues and the eigenvectors of the $N \times N$ covariance matrix. The window sizes for large images range from 4×4 to 10×10, leading to 16×16 to 100×100 covariance matrices. The calculations for the eigenvalues and eigenvectors of these large-sized matrices require a considerable amount of computation time and storage.

We are presenting in this paper a computationally short procedure for calculating the eigenvalues and eigenvectors of the covariance matrix. This simplification results from the bisymmetric properties of the covariance matrix, which is an outgrowth of using a square window as a sampling device over an image. We show that the eigenvalues and eigenvectors of the $N \times N$ bisymmetric covariance matrix can be obtained from the eigenvalues and eigenvectors of two $N/2 \times N/2$ submatrices. Since the eigenvector calculations are proportional to the third power of the matrix dimension, the proposed procedure reduces the computations by a factor of four.

II. Construction of the Sample Covariance Matrix

The entries in the covariance matrix of a given image are obtained by calculating the average covariance of all the elements in the image that has the same spatial relationship as the entry being considered. This procedure can best be illustrated using an example.

The 4×4 array shown in Fig. 1(a) represents a small image whose data are to be compressed. The image elements are labeled from 1 to 16. The size of the window for this example is 2×2 and the arrangement of components of the data vector X within each window is shown in Fig. 1(b). For this image, the average covariance array and the covariance matrix are computed as follows.

Each element in the average covariance array (Fig. 1(c)) is the average covariance of all the elements in the original image, having the same spatial relationship to each other as the element of the covariance array has to the lower center element. For example, the element c in the covariance array is located along the 45° diagonal line from the lower center (reference) element e. Accordingly, the entry c is the average covariance of 9 pairs of similarly spatially related image elements (5,2), (6,3), (7,4), (9,6), (10,7), (11,8), (13,10), (14,11), and (15,12). Other elements of the average covariance array are calculated using similar spatial relationships. The entries in the 4×4 covariance matrix (Fig. 1(d)) for the image can be obtained from the data contained in the average covariance array. For example, the entry at the second row, third column of the covariance matrix represents the covariance of x_2 and x_3. Elements x_3 and x_2 are along the 45° diagonal of the window and the element c in the average

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fields are estimated by conventional methods using typically the sample mean vector and sample covariance matrix. While these statistics of individual training fields are so computed, it is always desirable to estimate the mean vector and covariance matrix of the training class, which is the entirety of all the training fields. Also, as the characteristics of the fields vary along a flight line due to various reasons such as differences in atmospheric condition, temporal changes, and sun-angle effects, it is extremely important to update the statistics of the training class. This is achieved, of course, by augmenting the class with new training fields and deleting old training fields at the same time. This adaptive procedure thus calls for formulas to update the mean vector and covariance matrix. The following precisely derives such formulas.

II. Formulation of the Problem

Suppose a training class is given with K training fields

$$F_1 \equiv \{X_1, X_2, \cdots, X_{n_1}\}$$
$$F_2 \equiv \{X_{n_1+1}, X_{n_1+2}, \cdots, X_{n_1+n_2}\}, \cdots$$
$$F_K \equiv \{X_{n_1+n_2+\cdots+n_{K-1}+1}, \cdots, X_{n_1+n_2+\cdots+n_{K-1}+n_K}\}$$

where X_j is an r vector. Thus, the ith training field F_i consists of n_i elements, and the present training class consists of $N = n_1 + n_2 + \cdots + n_K$ elements. The mean vector (an r vector) and covariance matrix (an $r \times r$ matrix) of the class, μ and C, respectively, have been established[2] from the mean vectors $\{\mu_i\}$ and covariance matrices $\{C_i\}$ of the training fields, which have in turn been estimated by

$$\mu_i = \frac{1}{n_i} \sum_{X_j \in F_i} X_j \tag{1}$$

$$C_i = \frac{1}{n_i - 1} \sum_{X_j \in F_i} (X_j - \mu_i)(X_j - \mu_i)^T \tag{2}$$

where $\sum_{X_j \in F_i} X_j$ means

$$\sum_{j=n_1+n_2+\cdots+n_{i-1}+1}^{n_1+n_2+\cdots+n_{i-1}+n_i} X_j$$

and X^T denotes the transpose of X.

Now, the training class is augmented by l new training fields $F_{p_1}, F_{p_2}, \cdots, F_{p_l}$, where F_{p_i} consists of n_{p_i} elements and has mean vector μ_{p_i} and covariance matrix C_{p_i} as estimated similarly by (1) and (2). At the same time, m old training fields $F_{q_1}, F_{q_2}, \cdots, F_{q_m}$ are deleted from the class. The new training class thus consists of $K + l - m$ fields, totaling

$$\hat{N} \equiv N + \sum_{i=1}^{l} n_{p_i} - \sum_{i=1}^{m} n_{q_i}$$

elements.

Problem: Using the already computed estimates μ, C, $\{\mu_i\}$, and $\{C_i\}$, $i = 1, \cdots, K, p_1, \cdots, p_l$, find $\hat{\mu}$ and $\hat{C}$, where

$$\hat{\mu} \triangleq \frac{1}{\hat{N}} \sum_j{}' X_j \tag{3}$$

$$\hat{C} \triangleq \frac{1}{\hat{N} - 1} \sum_j{}' (X_j - \hat{\mu})(X_j - \hat{\mu})^T \tag{4}$$

where

$$\sum_j{}' X_j \equiv \sum_{\substack{X_j \in F_1, \cdots, F_K, F_{p_1}, \cdots, F_{p_l} \\ X_j \notin F_{q_1}, \cdots, F_{q_m}}} X_j \tag{5}$$

$\hat{\mu}, \hat{C}$ being recognized as the estimate of the mean vector and covariance matrix of the updated training class.

III. Solution

The formulas for $\hat{\mu}$ and $\hat{C}$ are

$$\hat{\mu} = \left(\frac{N}{\hat{N}}\right) \mu + \sum_{i=1}^{l} \left(\frac{n_{p_i}}{\hat{N}}\right) \mu_{p_i} - \sum_{i=1}^{m} \left(\frac{n_{q_i}}{\hat{N}}\right) \mu_{q_i} \tag{6}$$

$$\begin{aligned}\hat{C} = {} & \left(\frac{N-1}{\hat{N}-1}\right) C + \sum_{i=1}^{l} \left(\frac{n_{p_i}-1}{\hat{N}-1}\right) C_{p_i} - \sum_{i=1}^{m} \left(\frac{n_{q_i}-1}{\hat{N}-1}\right) C_{q_i} \\ & + \left(\frac{N}{\hat{N}-1}\right) \mu\mu^T + \sum_{i=1}^{l} \left(\frac{n_{p_i}}{\hat{N}-1}\right) \mu_{p_i}\mu_{p_i}^T \\ & - \sum_{i=1}^{m} \left(\frac{n_{q_i}}{\hat{N}-1}\right) \mu_{q_i}\mu_{q_i}^T - \left(\frac{\hat{N}}{\hat{N}-1}\right) \hat{\mu}\hat{\mu}^T. \end{aligned} \tag{7}$$

Proof:

$$\begin{aligned}\hat{\mu} &= \frac{1}{\hat{N}} \left\{ \sum_{X_j \in F_1, \cdots, F_K} X_j + \sum_{X_j \in F_{p_1}, \cdots, F_{p_l}} X_j - \sum_{X_j \in F_{q_1}, \cdots, F_{q_m}} X_j \right\} \\ &= \left(\frac{N}{\hat{N}}\right) \mu + \sum_{i=1}^{l} \left(\frac{n_{p_i}}{\hat{N}}\right) \mu_{p_i} - \sum_{i=1}^{m} \left(\frac{n_{q_i}}{\hat{N}}\right) \mu_{q_i}. \end{aligned}$$

From (2),

$$\hat{C}_i = \frac{1}{n_i - 1} \sum_{X_j \in F_i} X_j X_j^T - \frac{n_i}{n_i - 1} \mu_i \mu_i^T.$$

Thus,

$$\sum_{X_j \in F_i} X_j X_j^T = (n_i - 1) C_i + n_i \mu_i \mu_i^T. \tag{8}$$

Thus, from (4) and (8),

$$\begin{aligned}\hat{C} &= \frac{1}{\hat{N}-1} \sum_j{}' X_j X_j^T - \frac{\hat{N}}{\hat{N}-1} \hat{\mu}\hat{\mu}^T \\ &= \frac{1}{\hat{N}-1} \left\{ \sum_{X_j \in F_1, \cdots, F_K} X_j X_j^T + \sum_{X_j \in F_{p_1}, \cdots, F_{p_l}} X_j X_j^T - \sum_{X_j \in F_{q_1}, \cdots, F_{q_m}} X_j X_j^T \right\} - \frac{\hat{N}}{\hat{N}-1} \hat{\mu}\hat{\mu}^T \\ &= \left(\frac{N-1}{\hat{N}-1}\right) C + \sum_{i=1}^{l} \left(\frac{n_{p_i}-1}{\hat{N}-1}\right) C_{p_i} - \sum_{i=1}^{m} \left(\frac{n_{q_i}-1}{\hat{N}-1}\right) C_{q_i} \\ &\quad + \left(\frac{N}{\hat{N}-1}\right) \mu\mu^T + \sum_{i=1}^{l} \left(\frac{n_{p_i}}{\hat{N}-1}\right) \mu_{p_i}\mu_{p_i}^T \\ &\quad - \sum_{i=1}^{m} \left(\frac{n_{q_i}}{\hat{N}-1}\right) \mu_{q_i}\mu_{q_i}^T - \left(\frac{\hat{N}}{\hat{N}-1}\right) \hat{\mu}\hat{\mu}^T. \end{aligned}$$

IV. Special Case

Consider the case where l training fields are selected to make up a training class. $\{\mu_i\}$ and $\{C_i\}$, $i = 1, \cdots, l$, are the estimated mean vectors and covariance matrices of the l fields. To calculate the composite statistics $\hat{\mu}$ and $\hat{C}$ of the class from $\{\mu_i\}$ and $\{C_i\}$, it is noted that this is a special case of (6) and (7) when $\mu = 0$,

[2] Imagine that this is one of those generic iterations in a mathematical induction process where the μ and C are known, and the new $\hat{\mu}$ and $\hat{C}$ are going to be computed. Actual formulas for μ and C as expressed in terms of $\{\mu_i\}$ and $\{C_i\}$, $i = 1, \cdots, K$, are found in Section IV.

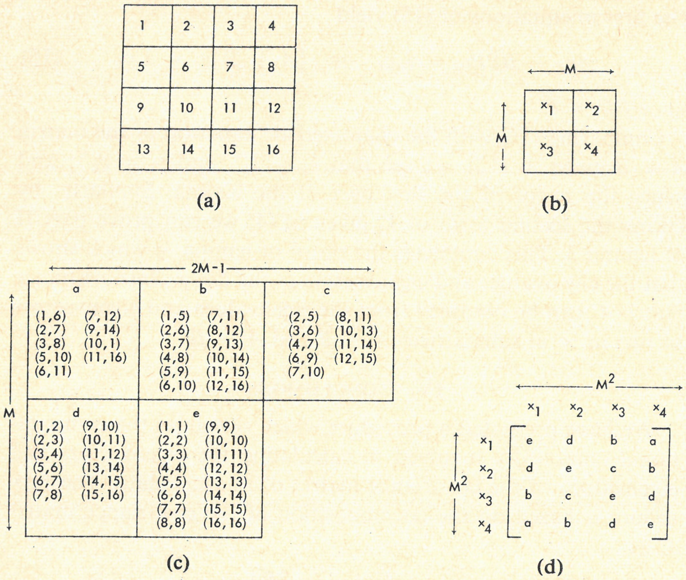

Fig. 1. (a) Original image. (b) Arrangement of variables within window. (c) Average covariance array. (d) Covariance matrix.

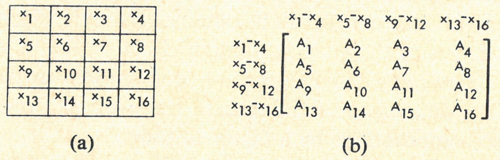

Fig. 2. (a) Elements of X in 4×4 window. (b) Partitioned form of covariance matrix C of X.

covariance array bears the same relationship to the lower center (reference) element e. Hence, the entry c is used to fill in the (2,3) element in the covariance matrix. Similarly, the remaining entries in the covariance matrix are obtained.

The procedure described in the preceding paragraphs can be used for square windows of any size $M \times M$, with M less than the overall dimension of the image itself. In the following we will restrict our attention to windows of size $M \times M$, where M is even. The sample covariance matrix obtained from the average covariance array has a bisymmetric[1] form. Also, the $M^2 \times M^2$ covariance matrix consists of M submatrices of dimension $M \times M$. These submatrices appear in a bisymmetric form within the covariance matrix. Let us consider the following example to further illustrate the bisymmetric properties of C.

Fig. 2(a) shows the arrangement of the components of X within a 4×4 window, and the 16×16 covariance matrix of X is shown in a partitioned form in Fig. 2(b). The 4×4 submatrices A_i represent the covariance of the elements of one row of the window with the elements of another row of the window. The matrices A_1, A_6, A_{11}, and A_{16} represent the covariance of the elements of a row with the elements of the same row. The spatial relationships existing between the elements of row 1 is the same as the spatial relationships between the elements of row 2, row 3, or row 4. Since the entries in all these matrices are obtained from the average covariance array using the spatial relationship between the elements, A_1, A_6, A_{11}, and A_{16} are identical. Next, let us consider the matrices A_2, A_7, A_{12}, A_5, A_{10}, and A_{15}, which represent the covariance between the element of one row of the window and the elements of an adjacent window. For instance, A_2 represents the covariance between the elements x_1,x_2,x_3,x_4 and x_5,x_6,x_7,x_8. Referring to Fig. 2(a), the spatial relationships between x_1,x_2,x_3,x_4 and x_5,x_6,x_7,x_8 and the spatial relationships between x_5,x_6,x_7,x_8 and x_9,x_{10},x_{11},x_{12} are the same. Hence, the entries in A_2 and A_7 will be the same. Extending this reasoning, it is easy to see that A_2, A_7, A_{12}, A_5, A_{10}, and A_{15} are identical. Similarly, it can be shown that A_3, A_8, A_9, and A_{14} are the same and that A_4 and A_{13} are also identical. Hence, the form of the covariance matrix becomes

$$C = \left[\begin{array}{cc|cc} A_1 & A_2 & A_3 & A_4 \\ A_2' & A_1 & A_2 & A_3 \\ \hline A_3' & A_2' & A_1 & A_2 \\ A_4' & A_3' & A_2' & A_1 \end{array}\right]$$

where A_i' denotes the transpose of A_i. Thus, the bisymmetric property of the covariance matrix is inherent in the window image sampling process. This example also illustrates that the $M^2 \times M^2$ covariance matrix consists of M submatrices of dimension $M \times M$ arranged in a bisymmetric form. The submatrices are not symmetric; however, for each matrix, $a_{ij} = a_{M+1-i,M+1-j}$.

The symmetry properties of the covariance matrix lead to a computationally simple procedure for the eigenvalue–eigenvector calculations. The procedure we will develop in the next section is similar to a method given by Ray and Driver [6] for decomposition of the Karhunen–Loève series representation of stationary random process.

III. Eigenvalues and Eigenvectors of the Covariance Matrix

We now show that the eigenvalues and eigenvectors of the $2m \times 2m$ covariance matrix C can be obtained by calculating the eigenvalues and eigenvectors of two $m \times m$ submatrices of C. This simplification results from the bisymmetry properties of C and the simplified procedure is developed through lemmas 1–3.

Lemma 1

The covariance matrix C, of dimension $2m \times 2m$, can be partitioned into $m \times m$ submatrices of the following form:

$$C = \left[\begin{array}{c|c} A & BP \\ \hline PB & A \end{array}\right]$$

where A and B are $m \times m$ submatrices of C, and P is an $m \times m$ matrix with ones along the NE–SW diagonal and zeros elsewhere; i.e., the (i,j)th element of P is given by

$$(P)_{ij} = \begin{cases} 1, & \text{for } j = m+1-i \\ 0, & \text{otherwise.} \end{cases}$$

The proof of lemma 1 follows from the construction of C.

Lemma 2

The eigenvectors V_i, $i = 1,\cdots,2m$, of C have either one of the following two forms:

$$V_i = \begin{bmatrix} v_i \\ Pv_i \end{bmatrix} \quad \text{or} \quad V_i = \begin{bmatrix} v_i \\ -Pv_i \end{bmatrix} \tag{4}$$

where v_i is an $m \times 1$ column vector.

Proof: The characteristic equation of C is given by $CY = \lambda Y$, where Y is an eigenvector of C corresponding to the eigenvalue λ. We want to prove that the ith component of Y, denoted y_i, satisfies

$$y_i = \pm y_{2m+1-i}. \tag{5}$$

[1] *Bisymmetric:* An $N \times N$ matrix A is bisymmetric if and only if $a_{ij} = a_{ji}$, $i,j = 1,2,\cdots,N$ and $a_{ij} = a_{N+1-i,N+1-j}$, $i,j = 1,2,\cdots,N$. It follows from this definition that $a_{ij} = a_{ji} = a_{N+1-i,N+1-j} = a_{N+1-j,N+1-i}$, $i,j = 1,2,\cdots,N$.

We begin by writing the characteristic equation in the form

$$\lambda y_i = \sum_{j=1}^{2m} c_{ij} y_j, \qquad i = 1,2,\cdots,2m \tag{6}$$

or

$$\lambda y_{2m+1-i} = \sum_{j=1}^{2m} c_{2m+1-i,j} y_j \tag{7}$$

where c_{ij} is the (i,j)th element of C. We may also write (7) as

$$\lambda y_{2m+1-i} = \sum_{j=1}^{2m} c_{2m+1-i,2m+1-j} y_{2m+1-j}, \qquad i = 1,2,\cdots,2m.$$

The bisymmetric property of C yields $c_{2m+1-i,2m+1-j} = c_{ij}$, and hence we may write the preceding equation as

$$\lambda y_{2m+1-i} = \sum_{j=1}^{2m} c_{ij} y_{2m+1-j}. \tag{8}$$

Equations (6) and (8) both represent $2m$ equations in $2m$ unknowns. Letting $y_{2m+1-i} = Z_i$, (8) becomes

$$\lambda Z_i = \sum_{j=1}^{2m} c_{ij} Z_j. \tag{9}$$

From (6) and (9) it is obvious that the solution for the y_i is also the solution for the Z_i, i.e., y_i and y_{2m+1-i} have the same solution.

Since the signs of the eigenvectors are not unique, forcing the norm of the eigenvectors to 1 makes $y_i = \pm y_{2m+1-i}$, $i = 1,\cdots,2m$. Thus we establish (4), and hence the proof of the lemma.

Lemma 3

The $2m$ eigenvalues $\lambda_1,\lambda_2,\cdots,\lambda_{2m}$ and the corresponding eigenvectors $V_1,V_2,\cdots,V_{2m}$ of C divide into the following two groups: part I,

$$\lambda_i = \lambda_i^+ \qquad V_i = \begin{bmatrix} v_i^+ \\ Pv_i^+ \end{bmatrix}, \qquad i = 1,2,\cdots,m \tag{10}$$

and part II,

$$\lambda_i = \lambda_i^- \qquad V_{i+m} = \begin{bmatrix} v_i^- \\ -Pv_i^- \end{bmatrix}, \qquad i = 1,2,\cdots,m \tag{11}$$

where λ_i^+ and v_i^+ are the eigenvalues and eigenvectors[2] of the $m \times m$ submatrix $A + B$ and λ_i^- and v_i^- are the eigenvalues and eigenvectors of the $m \times m$ submatrix $A - B$; i.e.,

$$[A + B]v_i^+ = \lambda_i^+ v_i^+ \tag{12}$$

$$[A - B]v_i^- = \lambda_i^- v_i^- \tag{13}$$

where $i = 1,2,\cdots,m$.

Proof: The characteristic equation of C is

$$CV_i = \lambda_i V_i.$$

Using the partitioned form of C, we may write the preceding equation as

$$\begin{bmatrix} A & BP \\ PB & A \end{bmatrix} V_i = \lambda_i V_i. \tag{14}$$

Lemma 2 gives two forms of V_i, and substituting the first form given in (4), we can write (14) as

$$\begin{bmatrix} A & BP \\ PB & A \end{bmatrix} \begin{bmatrix} v_i \\ Pv_i \end{bmatrix} = \lambda_i \begin{bmatrix} v_i \\ Pv_i \end{bmatrix}. \tag{15}$$

[2] The vectors v_i^+ and v_i^- are normalized to give $\|v_i^+\|^2 = \|v_i^-\|^2 = \frac{1}{2}$ so that $\|V_i\|^2 = 1$.

The first m equations of (15) yield

$$Av_i + BPPv_i = \lambda_i v_i \quad \text{or} \quad [A + B]v_i = \lambda_i v_i \tag{16}$$

since $PP = I$.

Comparing (16) with the characteristic equation of the matrix $A + B$ given in (12), we see that $\lambda_i = \lambda_i^+$ and $v_i = v_i^+$; and hence the proof of the first part of the lemma. Similarly, by taking the second form of V_i given in (4) we can prove part II of lemma 3. This completes the proof of lemma 3, which shows that the eigenvalues and eigenvectors of the $2m \times 2m$ covariance matrix C can be obtained from the eigenvalues and eigenvectors of two $m \times m$ submatrices $A + B$ and $A - B$.

IV. Conclusions

We have presented a procedure which simplifies the computational complexity involved in calculating the eigenvectors and eigenvalues of the covariance matrix of imagery data. The procedure is based on the decomposition of the covariance matrix C as

$$C = \left[\begin{array}{c|c} A & BP \\ \hline PB & A \end{array}\right].$$

We have shown that the eigenvalues and eigenvectors of the $2m \times 2m$ covariance matrix can be obtained from the eigenvalues and eigenvectors of the $m \times m$ submatrices $A + B$ and $A - B$. Since the eigenvector calculations are proportional to the third power of the dimension of the matrix, the proposed procedure reduces the computations by a factor of four.

Also, the eigenvectors of the covariance matrix has to be stored or transmitted to the receiver for reconstructing the imagery data according to (1). The symmetry property of the eigenvectors given in (10) and (11) enables us to store or transmit only half of the components of the eigenvectors. This results in a considerable saving in transmission time, especially if the dimension of the eigenvector is large.

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On User Supplied Evaluations of Time-Shared Computer Systems

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Abstract—**Comments are made regarding the collection of user preference data for varying characteristics of time-sharing systems. These "utility functions," when determined for a number of variables, can be used as an aid to managers and designers of time-sharing service facilities.**

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