# USING ADAPTIVE SYSTEMS FOR MODELING SOCIOECONOMIC SYSTEMS

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#### Abstract

The research presented herein is reporting the accomplishments in the area of structure determination using the theory of automatons or sequential machines. A thorough discussion is undertaken concerning adaptive systems with a controlling automaton. A dynamic socioeconomic system is presented with varying control and feedback functions. An automaton controller is derived with system states determining structure of the automaton. With the structure derived, future research is discussed which would allow theoretical reduction/construction of the controller.

The system model chosen is a population model describable by differential or difference equations. The system model along with the adaptive controller presents a new approach to modeling socio-economic systems.

### I. Introduction

Methods for determining structure for socioeconomic systems have been proposed by many authors (1, 2, 3, 4). These methods are used in both theoretical and applied models of phenomena found within socioeconomic systems.

One of the most utilized methods of structure for economic systems is that based on the work of Leontief [1]. Using Leontief matrices, an input/output structure is derived which utilizes constant coefficients in a matrix array to project industrial output given a proportional mix of the necessary inputs. While this structure has proved invaluable in short run forecasting of economic conditions, better dynamic models with varying controls or feedback loops and nondeterministic variables must be developed. The phenomena encountered when modeling socioeconomic systems presents considerable difficulty to any socioeconomic system theorist and presents a clear mandate to the same for better structure determination and more realistic models.

Other structures have been examined by John Warfield [2]. While Warfield's theories are complex and contribute much to understanding model structures their applicability to real world problems must still be demonstrated. The modeling done by J. W. Forrester [3] is indeed complex, far sweeping in its implications drawn from actual simulations but weak in terms of justification of structure variables used throughout the model.

Structure determination for complex socio-economic processes has also been examined by Hogan [4]. In Hogan's research a method has been outlined which uses probablistic

models with direct and alternative paths for structure determination. The structures for these probablistic path models have advanced the modeling of socioeconomic systems; however, more work is being done by the author at this time in this area to obtain a complete modeling methodology and incorporate these models within the context of adaptive controllers.

The research presented here is a new attempt and part of an ongoing research effort to clarify the area of structure determination. The method used here is derived from the theory of automata or sequential machines.

Modeling of any real world system entails (1) creating a model structure, (2) determining specific parameters for the model, and (3) simulating for predictive purposes.

This paper discusses one way of using adaptive structures for modeling. We suggest that using an automaton to select which one of a set of relations between inputs and outputs is operating yields an effective adaptive structure for modeling socio-economic systems. The transition function of the controlling automaton can be determined from specific intuitive or theoretical considerations about the system being modeled or from an empirically observed data sequence.

Section II is a duscission of the adaptive system. Section III discusses the system identification problem and Section IV gives an demographic example of an adaptive system. Conclusions and recommendations are presented in Section V

# II. An Adaptive System

One expectation which is almost sure to be fulfilled in the system modeling situation is the fact that in complex systems there probably occurs pairs of subsystems tightly coupled one to another. When the direction of this coupling is one way it is sometimes possible to view one subsystem as the controller (not in the control theory sense) for another. In this case the pair of subsystems could be called adaptive.

In what way does an adaptive system manifest itself to an observer? The system seems to have a pattern in the way its dynamics seem to change. There seems to be a changing relation between the input and output and the way the relation changes seems to be highly structured. The output may appear as if it is trying to achieve a certain value or certain cycle. In time it seems to do better and better at what it

tries to achieve. We want to say that system is "learning" or we want to say that the input/output relations are changing.

Such anthropomorphic characteristics as "learning," "adaptive," and "self-organizing" are used to describe this situation. If we used these terms to describe a system, we must mean that something inside is changing. What is changing is the relationship between the output and the input. The change must be caused by the input since the model has no other causes. When we say that the relationship between the output and the input is changing, we necessarily must have a particular system decomposition in mind. The system must have two parts: the first is the part which transforms the inputs to the outputs (this is the part which is changing) and the second is the part whose function is in control of the input/output transfer functions (this part does not change). We are thinking of systems like that diagrammed in Figure 1.

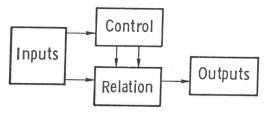


Figure 1. A Block Diagram of an Adaptive System.

We wish to describe a special case of the adaptive system. The systems we describe are built from relations between the input and possible outputs, with a multiplexer switch selecting the system output from the set of possible outputs and a controller consisting of an automaton. Figure 2 shows a typical system.

A controlling automaton can be specified in several ways:

(1) entirely empirical by observing the input/output data sequence of the controller and determining a reduced input/output equivalent automaton;

(2) entirely intuitive by designing a controlling automator satisfying given theoretical constraints and agreeing with expected behavior;

(3) part empirical and part intuitive.

Section III discusses several possibilities for the identification of the transition function of the adaptive controller using the observed input/output data sequence.

#### III. System Identification

Identifying the controlling automaton can begin with the knowledge of the input set of the controlling automaton and the possible system input/output relation among which the automaton selects or can begin with only knowledge of the input set of the controlling automaton. Section III.1 discusses the first case and Section III.2 discusses the second case.

III.1 Identification of Controlling Automaton Transition

Once the input set of the controlling automaton is defined and the possible system input/output relations are also

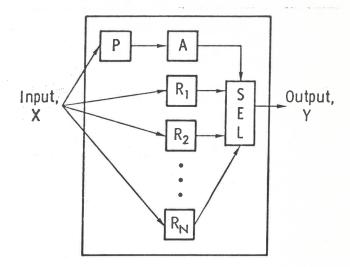


Figure 2. An Example of an Adaptive System Where:

R<sub>1</sub>, R<sub>2</sub>, ..., R<sub>N</sub>

SEL

Is a Selector Switch
Controlled by A

A

Is the Controlling
Automaton

P

Is a Preprocessor
for A

defined, its sequential behavior of the controlling automaton needs to be determined by observing examples of its input/output behavior. We assume that it is a discrete system. It has discrete attributes, inputs, outputs, time units, and discrete internal states. We consider the output of the controlling automaton to be its state. We consider the discrete system to be a black box and we wish to estimate what is inside it functionally. Normally, we cannot open the black box because it would be destroyed or its operation would be disturbed. However, we may be allowed to observe its inputs and outputs. The identification problem is: given the observed sequence of inputs and outputs, determine the transition function of a simplest system which behaves in the manner described by the observed input/output sequence.

The identification of transition function is a synthesis problem and can be attacked by considering a length N input sequence and its corresponding output sequence as defining a chain sequential machine having N states. When the  $n^{th}$  input is applied, the machine is in the  $n^{th}$  state. It transits to the (n+1)-state and outputs the  $n^{th}$  output. If any input other than the  $n^{th}$  input were to be applied to the chain sequential machine when it is in the  $n^{th}$  state, both output and next state are not defined. In essence, the chain sequential machine has exactly the information its defining input and output sequence has.

To determine a functionally input-output equivalent but reduced machine, one need only apply the state minimization procedures for incompletely specified machines. Unfortunately, the techniques which have been published involve too

much memory for a machine with 1000 states and 10 binary variables for the input variable set. Therefore, we must formulate algorithms where memory space is proportional to number of states and which may not yield minimal machines but yield close to minimal machines.

Beginning with Aufenkamp and Hohn [5], a number of state minimizing algorithms have appeared in the literature for deterministic and completely specified machines [6, 7, 8]. Other algorithms for finding maximum compatible state sets for incompletely specified machines have appeared [9, 10, 11]. Algorithms for constructing probabilistic machines have also appeared [12, 13, 14]. The approach that we must take in state minimizing is just slightly different than ones which are already known. First, the reduction which we need to do in the systems context begins with a chain sequential machine. This should let the algorithm not need more than an memory locations where n is the length of the initial input-output string. Second, the incompletely specified condition of the chain sequential machine is stronger than for the general incompletely specified machine: In the chain sequential machine if a transition is allowed, then there must be a concomitant output also. Finally, a careful analysis of the state merging process should show that a lot of consistency checking can be eliminated by performing the right merges first.

Another fact which must be considered in identifying a system transition function is the noise which corrupts both the observed input and output strings. As shown in Figure 3 the observed strings are the corrupted version of the actual input and output strings. This corruption will undoubtedly make any system transition function based on the observed strings more complicated than the actual system transition function. This motivates the following method to simplify the transition structure: determine those inputs or outputs which if changed to other values would most reduce the number of states in a deterministic machine. According to Judea Pearl, who is studying complexity, (personal communication) if there is really structure there, deliberate modification of a few input or output values would considerably reduce the number of states in the minimal state input-output equivalent machine. However, if the input-output strings come from a machine with no structure, then modification of a few input or output values will hardly change the number of states in the minimal machine.

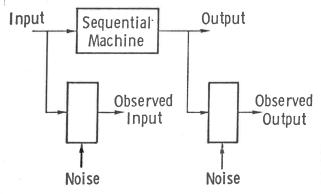


Figure 3. Shows How In Actual Observation
Noise Corrupts Both the Observed
Input and Observed Output Strings.

# III.2 Identification of Controlling Automaton And System Input/Output Relations

There are several classical methods for identifying the relations between the input and output continuous systems. If the system, in this case each relation, is available for testing, then an impulse forcing function can be applied to the input. The observed output response can be used, through Fourier analysis, to determine the transfer relation [15]. Davies [16] considers the case when the system is not available for testing offline. He presents a method for deducing the transfer relation using maximum length noise sequences and cross-correlation between the input and output signals. Davies goes on to describe an online, almost real time system identification computer. More complex problems can be solved with techniques developed by Kagiwada [17]. These include quazi-linearization, dynamic programming and invariant imbedding.

Starke [8] worked on similar identification problems for sequential machines. His results are restricted in the sense that one must be given a weakly initial machine. These methods identify the initial state of the machine. Starke also uses a second machine, similar in structure to Davies' computer, to perform the identification.

The problem we are faced with for the adaptive system is that the automaton is selecting a different input-output relation over time. We may not be able to associate the output with any one relation. The same input/output pair may arise from two different states of the controlling automaton.

The use of additional machines introduced by Davies and Starke is most interesting. We can develop a machine that would execute the following program:

(1) observe the input and output signals,

estimate the possible states the automata might be in,
 add this new input output information to the information accumulated for the relations associated with the possible states of the automaton,

 produce an output based on current knowledge of the system,

(5) compute a performance index based on the observed output signal and the produced output,

(6) modify the relations and/or automaton to maximize the performance index, and

(7) go to step (1).

The procedure is essentially the maximizing of the performance index over the domain of possible automatons and relations. To ease our task, we can restrict the domain by limiting the number and type of relations and the size of the controlling automaton.

#### IV. Population Model

A model for the population process is presented in Figure 4. The population model in Figure 4 which we used to demonstrate our research is a closed nonproliferating system. X 1, X2 represent the levels of population in the age groups (0-18), and (18-100), respectively. The variable  $\alpha$ , is the normal transition which occurs between the age groups based on the aging process while  $\mu_1$ ,  $\mu_2$  are the death rates for the age groups. The birth rate is represented by the variable "b" and will be controlled by an automaton with states dependent upon socioeconomic conditions which produce different birth rates.

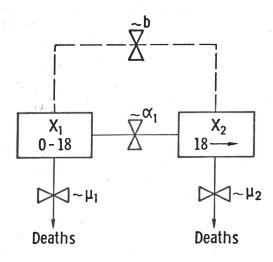


Figure 4. Population Model.

The equation for the population model are:

$$\dot{x}_{1} = -\alpha_{1}X_{1} - \mu_{1}X_{1} + bX_{2}$$

$$\dot{x}_{2} = -\mu_{2}X_{2} + \alpha_{1}X_{1}$$

Incorporating the death rate  $\mu_1$  in with the age transition al and employing an integral approximation for the non-uniform population in X1, we have,

$$\alpha_1 = \frac{1}{10} (1 - \mu_1/2)$$

Therefore the model becomes

$$\dot{X}_{1} = -\frac{1}{18} (1 - \mu_{1}/2) X_{1} + b X_{2}$$

$$\dot{X}_{2} = -\mu_{2} X_{2} + \alpha_{1} X_{1}$$

We specify the automaton to control the birth rates within the model based on three states or possible conditions in time. These states are:

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2.	Zero birth rate	(ZBR)
	Escalated birth rate	(EBR)
4.	Normal experienced birth rate	(NBR)
	with growth	

A4/1

The birth rates during war necessarily must decline because of the decrease in males. Before and after war, it is very common to have a normal birth rate with growth and escalated birth rate, respectively. The zero birth rate is a condition imposed by society through the media to avert possible socio-economic disasters such as shortages of food, commodities, etc. and has occurred in the United States over the last ten years. Constructing an automaton around these intuitive, yet probable hypotheses, we derive, based on the state of the automaton and the input to the automaton from the population model,

		Ing	out						
State		1	2	3	4	5	6		
War	а	a	а	а	c,d	а	a		
ZBR	b	b	b	c,d	c,d	b	ű		
EBR	С	Ь	С	С	Ь	С	а		
NBR	d	Ь	d	d	d	Ь	а		
Table 1. State/Input Table									

where the inputs one through six are defined as:

- 1. Level of population has reached twice the norm
- Normal level of population Below the normal level of population
- 4. Time span of five years
- 5. Time span of ten years
- 6. Time span of twenty years.

Constructing the automata which will control the birth rate by using Table 1, we have,

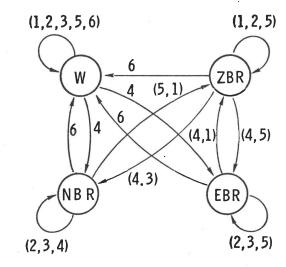


Figure 5. Birth Rate Automaton Controller

Now we must relate the population model to the automaton and examine the possibility of reduction of the birth rate control. Redrawing the system model, we construct four feedback paths for the birth rate and state the computer algorithms for utilization of the alternate paths. The population model is represented in discrete time equations and diagrammed with alternate birth rates (Figure 6). The controller automaton for the population model is shown in Figure 5.

The variable B for birth rate is in states a,b,c, and are dependent upon the conditions, within the model specified as inputs 1 through 6.

The six inputs determined by system conditions which specified the state transitions are:

- 1. excessive level above the normal population level
- normal population level
- below normal population level
- 4. time span of five years
- 5. time span of ten years
- 6. time span of twenty years

This model and adaptive controller demonstrate the techniques outlined previously within the paper. Further work has demonstrated the feasibility of such an approach and facilitates system reduction from theory found when dealing with sequential machines.

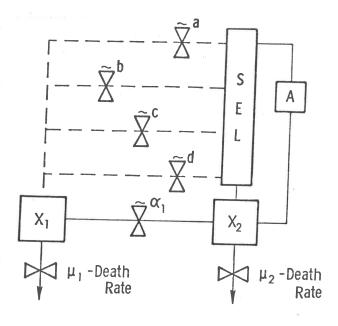


Figure 6. Population Model with Automaton (A) Controller

$$\begin{split} X_1(k+1) &= \left[1 - \frac{\Delta T}{18} (1 - \mu_1/2)\right] \chi_1(k) + \underline{B} \ \chi_2(k) \\ X_2(k+1) &= \left[1 - \frac{\Delta T}{18} \mu_2\right] X_2(k) + \frac{\Delta T}{18} (1 - \mu_1/2) \ \chi_1(k) \end{split}$$

## V. Conclusions and Recommendations

The adaptive systems model presented here along with the automaton controller has demonstrated the feasibility of constructing adaptive socioeconomic models. The methodology derived from these adaptive models facilitates a reduction algorithm heretofore used when looking at sequential machines.

Within an ongoing research effort, the problem of specifying the controller given the input/output system information is occurring. Future research should indicate many advantages using the concepts advanced within this paper.

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