TOPOGRAPHIC CLASSIFICATION OF DIGITAL IMAGE INTENSITY SURFACES

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Abstract

A complete mathematical treatment is given for describing the topographic classification for the underlying grey tone intensity surface of a digital image. Each picture element is independently classified into a unique descriptive label from the set { peak, pit, ridge, ravine, saddle, flat, slope, convex hill, concave hill, and saddle hill } based on the first and second directional derivatives of the estimated image intensity surface. A local facet model two-dimensional cubic polynomial fit is done to estimate the image intensity surface.

1.0 Introduction

Describing the fundamental structure of a digital image is a primary problem encountered in any computer vision system. Information in an image is captured in the form of its light intensity variations. These light intensity variations must be used to explain the orientation and reflectance of the surfaces on the objects being imaged. We believe that the classification of the image intensity surface into a complete set of topographic types will be a definite aid in this regard.

The facet model states that all processing of digital image data has its final authoritative interpretation relative to what the processing does to the underlying grey tone intensity surface. The digital image pixel values are noisy sampled observations of the underlying surface. Thus, in order to do any processing, we at least have to estimate at each pixel position what this underlying surface is. This requires a model which describes what the general form of the surface would be in the neighborhood of any pixel if there were no noise. To estimate the surface from the neighborhood around a pixel then amounts to estimating the free parameters of the general form. It is important to note that if a different general form is assumed, then a different estimate of the surface is produced. Thus the assumption of a particular general form is necessary and has consequences.

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The general form we use is a bivariate cubic. We assume that the neighborhood around each pixel is suitably fit by a bivariate cubic. Having estimated this surface around each pixel, the first and second directional derivatives are easily computed by analytical means. The topographic classification of the surface facet is totally based on the first and second directional derivatives. We classify each surface point as peak, pit, ridge, ravine, saddle, flat, slope, convex hill, concave hill, or saddle hill. Our set of ten topographic labels is complete in the sense that they are able to describe any topographic structure which may occur on a digital image intensity surface.

1.1 Previous Work

Detection of topographic structures in a digital image is not a new idea. There have been a wide variety of techniques described to detect pits, peaks, ridges, ravines, and the like.

Lee and Fu (1981) define a set of 3X3 templates which they convolve over the image to give each class, except plain, a figure of merit. plain. Their set of labels include {none, plain, slope, ridge, valley, foot, shoulder}. Thresholds are used to determine into which class the pixel will fall. In their scheme a pixel may satisfy the definition of zero, one, or more than one class. Ambiguity is resolved by choosing the class with the highest figure of merit.

Toriwaki and Fukumura (1978) take a totally different approach from all the others. They use two local features of grey-level pictures, connectivity number and coefficient of curvature, for classification of the pixel into {peak, pit, ridge, ravine, hillside, pass}. They then describe how to extract structural information from the image once the labelings have been made. This structural information consists of ridge-lines, ravine-lines, and the like.

Grender's (1976) algorithm compares the grey level elevation of a central point with surrounding elevations at a given distance around the perimeter of a circular window and the radius of the window may be increased in successive passes through the image. His topographic labeling set consists of {slope, ridge, valley, knob, sink, saddle}.

Paton (1975) uses a six term quadratic expansion in Legendre polynomials fitted to a small disk around each pixel. The most significant coefficients of the second order polynomial yield a descriptive label chosen from the set {constant, ridge, valley, peak, bowl, saddle, ambiguous}. He uses the continuous least squares fit formulation in setting up the surface fit equations as opposed to the discrete least squares fit used in the facet model. The continuous fit is a more expensive computation than the discrete fit and results in a step-like approximation.

Johnston and Rosenfeld (1975) attempt to find peaks by finding all points P such that no points in an n-by-n neighborhood surrounding P have greater elevation than that of P. Pits are found in an analagous manner. To find ridges, they identify points that are either east-west or north-south elevation maxima. This is done using a 'smoothed' array in which which each point is given the highest elevation in a 2X2 square containing it, and on this array also, east-west and north-south maxima are found. Ravines are found in a similar manner.

Peuker and Johnston (1972) take a similar approach and characterize the surface shape by the sequence of positive and negative differences as successive surrounding points are compared to the central point. Peuker and Douglas (1975) describe several variations of this method for detecting one of the shapes {pit, peak, pass, ridge, ravine, break, slope, flat}. They start with the most frequent feature (slope) and proceed to the less frequent, thus making it an order-dependent algorithm.

1.3 A Mathematical Approach

From the previous discussion one can see that a wide variety of methods and labels were proposed to describe the topographical structure in a digital image. Some of the methods require multiple passes through the image while others may give ambiguous labels to a pixel. All these methods are heuristic in nature.

In this paper we describe a local, parallel, one-pass method in which each pixel is given a unique label from the set { peak, pit, ridge, ravine, saddle, flat, slope, convex hill, concave hill, and saddle hill } based on the first and second directional derivatives of the estimated grey tone intensity surface.

2.0 The Directional Derivative Pixel Classifier

Our classification approach is based on the estimation of the first and second order directional derivatives at each sampled point. We regard the digital picture function as a sampling of the underlying function f, where some kind of random noise is added to the true function values. To do this, our classifier must assume some kind of parametric form for the underlying function f, use the sampled brightness values of the digital picture function to estimate the parameters, and finally make decisions regarding the locations of relative extrema of partial derivatives based on the estimated values of the parameters. In Section 2.1 we discuss the directional derivative. In Section 2.2 we define the basis of the topographic classification in terms of the directional derivative. In Section 3 we give the details of the implementation of the classifier.

2.1 The Directional Derivative

In two dimensions, the rate of change of a function f depends on direction. We denote the directional derivative of f at the point (r,c) in the direction β by $f'_\beta(r,c)$. It is defined as

$$f'_\beta(r,c) = \lim_{h \to 0} \frac{f(r+h*\sin\beta, c+h*\cos\beta)}{h}$$

The direction angle β is the clockwise angle from the column axis. It follows directly from this definition that

$$f'_\beta(r,c) = \frac{\partial f}{\partial r}(r,c) * \sin\beta + \frac{\partial f}{\partial c}(r,c) * \cos\beta$$

We denote the second derivative of f at the point (r,c) in the direction β by $f''_\beta(r,c)$ and it follows that

$$f''_\beta = \frac{\partial^2 f}{\partial r^2}*\sin^2\beta + 2*\frac{\partial^2 f}{\partial r \partial c}*\sin\beta*\cos\beta + \frac{\partial^2 f}{\partial c^2}*\cos^2\beta$$

The gradient of f is a vector whose magnitude,

$$((\partial f/\partial r)^2 + (\partial f/\partial c)^2)^{1/2},$$

at a given point (r,c) is the maximum rate of change of f at that point, and whose direction,

$$\tan^{-1}((\partial f/\partial r) / (\partial f/\partial c)),$$

is the direction in which the surface has the greatest rate of change.

2.2 The Classification Scheme

We will use the following notation to describe the classification scheme. Let

∇f = Gradient vector of a function f

$||\nabla f||$ = Gradient magnitude

$\omega^{(1)}$ = direction in which second directional derivative has greatest magnitude

$\omega^{(2)}$ = direction in which second directional derivative has least magnitude

λ_1 = value of second directional derivative in the direction of $\omega^{(1)}$

λ_2 = value of second directional derivative in the direction of $\omega^{(2)}$

$\nabla f \cdot \omega^{(1)}$ = value of first directional derivative in the direction of $\omega^{(1)}$

$\nabla f \cdot \omega^{(2)}$ = value of first directional derivative in the direction of $\omega^{(2)}$

Without loss of generality, we assume $|\lambda_1| >= |\lambda_2|$.

Each type of topographic structure in our classification scheme is defined in terms of the above quantities. In order to calculate these values, the first and second order partials with respect to r and c need to be approximated. These five partials are listed below:

$\partial f/\partial r$, $\partial f/\partial c$, $\partial^2 f/\partial r^2$,

$\partial^2 f/\partial c^2$, $\partial^2 f/\partial r \partial c$

The gradient vector is simply $(\partial f/\partial r, \partial f/\partial c)$. The second directional derivatives may be calculated by forming the Hessian where the Hessian is a 2X2 matrix defined as:

$$H = \begin{vmatrix} \partial^2 f/\partial r^2 & \partial^2 f/\partial r \partial c \\ \partial^2 f/\partial r \partial c & \partial^2 f/\partial c^2 \end{vmatrix}$$

The eigenvalues of the Hessian are the values of the extrema of the second directional derivative, and their associated eigenvectors are the directions of the extrema. Thus,

$$H\omega^{(1)} = \lambda_1 \omega^{(1)} \text{ and } H\omega^{(2)} = \lambda_2 \omega^{(2)}.$$

Since H is a 2X2 symmetric matrix, calculation of the eigenvalues and eigenvectors can be done efficiently using the method of Rutihauser (1971). We may obtain the values of the first directional derivative in the direction of either extrema of the second directional derivative by simply taking the dot product of the gradient with the appropriate eigenvector:

$$\nabla f \cdot \omega^{(1)}$$

$$\nabla f \cdot \omega^{(2)}$$

Each category in the directional derivative classification scheme will be discussed in detail in sections 2.2.1 to 2.2.7.

2.2.1 Peak

A peak (knob) occurs where there is a local maxima in all directions. In other words, we are on a peak if no matter what direction we look in, we see no point that is as high as the one we are on as illustrated in Figure 1. The curvature is downward in all directions. At a peak the gradient is zero and the second directional derivative is negative in all directions. To test whether the second second directional derivative is negative in all directions, we just have to examine the value of the second directional derivative in the directions which make it smallest and largest. A pixel is therefore classified as a peak if the following conditions are true:

$$||\nabla f|| = 0,\ \lambda_1 < 0,\ \lambda_2 < 0.$$

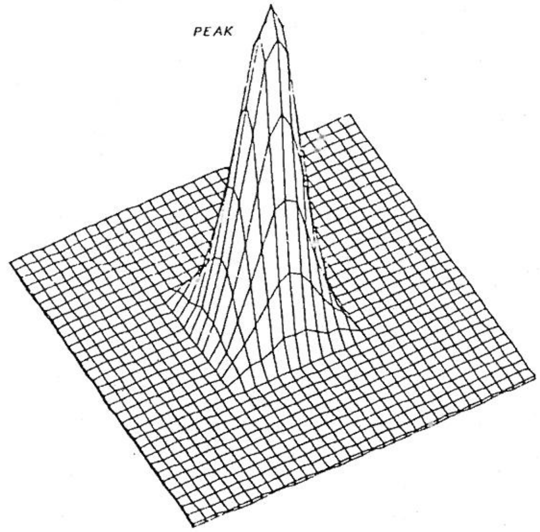

Figure 1: Peak Surface

2.2.2 Pit

A pit (sink, bowl) is identical to a peak except it is a local minima in all directions rather than a local maxima. At a pit the gradient is zero and the second directional derivative is positive in all directions. A pixel is classified as a pit if the following conditions are true:

$$||\nabla f|| = 0,\ \lambda_1 > 0,\ \lambda_2 > 0.$$

2.2.3 Ridge

A ridge occurs on a ridge-line, a curve consisting of a series of ridge points. As we walk along the ridge-line the pixels to the right and left of us are lower than the ones we are on. Furthermore, the ridge-line may be flat, sloped upward, sloped downward, curving upward, or curving downward. A ridge occurs where there is a local maximum in one direction as illustrated in Figure 2. It, therefore, must have negative curvature in that direction and also a zero directional derivative in that same direction. A pixel is classified as a ridge if any one of the following three sets of conditions are true:

$$||\nabla f|| = 0, \lambda_1 < 0, \lambda_2 = 0$$

or

$$||\nabla f|| \neq 0, \lambda_1 < 0, \nabla f \cdot \omega^{(1)} = 0$$

or

$$||\nabla f|| \neq 0, \lambda_2 < 0, \nabla f \cdot \omega^{(2)} = 0$$

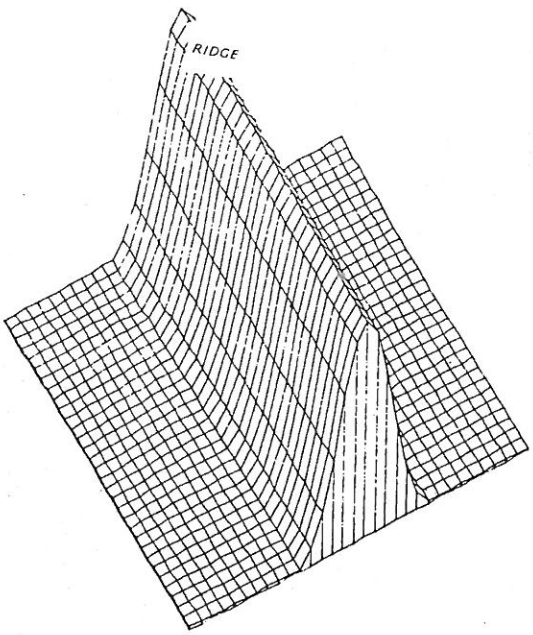

Figure 2: Ridge-Line

2.2.4 Ravine

A ravine (valley) is identical to a ridge except it is a local minimum in one direction rather than maximum. As we walk along the ravine-line, the pixels to the right and left of us are higher than the one we are on. A pixel is classified as a ravine if any one of the following three sets of conditions are true:

$$||\nabla f|| = 0, \lambda_1 > 0, \lambda_2 = 0$$

or

$$||\nabla f|| \neq 0, \lambda_1 > 0, \nabla f \cdot \omega^{(1)} = 0$$

or

$$||\nabla f|| \neq 0, \lambda_2 > 0, \nabla f \cdot \omega^{(2)} = 0$$

2.2.5 Saddle

A saddle occurs where there is a local maximum in one direction and a local minimum in a perpendicular direction as illustrated in Figure 3. It, therefore, must have positive curvature in one direction and negative curvature in a perpendicular direction. At a saddle the gradient magnitude must be zero and the extrema of the second directional derivative must have opposite signs. A pixel is classified as a saddle if the following conditions are true:

$$||\nabla f|| = 0, \lambda_1 * \lambda_2 < 0.$$

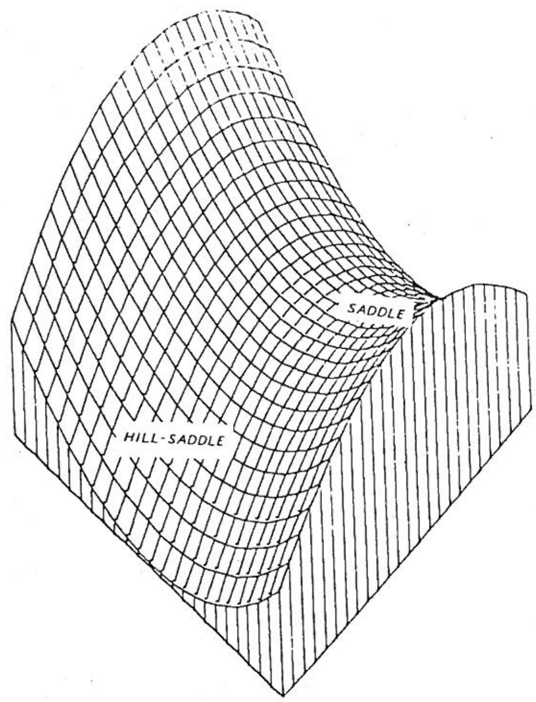

Figure 3: Saddle Surface

2.2.6 Flat

A flat (plain) is a simple, horizontal surface as illustrated in Figure 4. It, therefore, must have zero gradient and no curvature. A pixel is classified as a flat if the following conditions are true:

$$||\nabla f|| = 0, \lambda_1 = 0, \lambda_2 = 0$$

Given that the above conditions are true, a flat may be further classified as a foot or a shoulder. A foot occurs at that point where the flat just begins to turn up into a hill. At this point, the third directional derivative in the direction towards the hill will be nonzero and the surface increases in this direction. The shoulder is an analagous case and occurs where the flat is ending and turning down into a hill. At this point, the maximum magnitude of the third directional derivative is nonzero, and the surface decreases in the direction towards the hill. If the third directional derivative is zero in all directions then we are on a flat, not near a hill. Thus a flat may be further qualified as being a foot or shoulder, or not qualified at all.

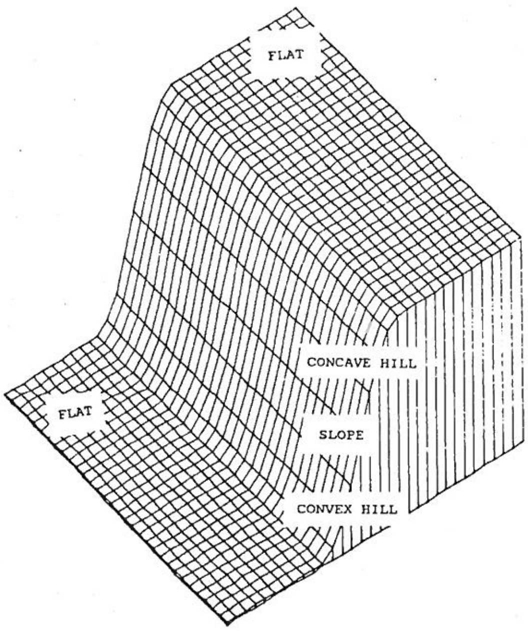

Figure 4: Sloped Surface

2.2.7 Hillside

A hillside is anything not covered by the previous categories. It has a non-zero gradient and no strict extrema in the directions of maximum and minimum second directional derivative. If the hill is simply a tilted flat (i.e. has constant gradient), we call it a slope. If its curvature is upward, we call it a convex hill. If its curvature is down, we call it a concave hill. If the curvature is up in one direction and down in a perpendicular direction, we call it a saddle hill. A saddle hill is illustrated in Figure 3, and the slope, convex hill, and concave hill are illustrated in Figure 4. A pixel is classified as a hillside if all the following conditions are true ('->' represents the operation of logical implication):

$$\lambda_1 = \lambda_2 = 0 \rightarrow ||\nabla f|| \neq 0$$

and

$$\lambda_1 \neq 0 \rightarrow \nabla f \cdot \omega^{(1)} \neq 0$$

and

$$\lambda_2 \neq 0 \rightarrow \nabla f \cdot \omega^{(2)} \neq 0$$

We can differentiate between the different types of hillsides by the values of the extrema of the second directional derivative. The distinction can be made as follows:

SLOPE if $\lambda_1 = \lambda_2 = 0$,

CONVEX if $\lambda_1 >= \lambda_2 >= 0$, $\lambda_1 \neq 0$,

CONCAVE if $\lambda_1 <= \lambda_2 <= 0$, $\lambda_1 \neq 0$,

SADDLE if $\lambda_1 * \lambda_2 < 0$.

A slope may occur where the curvature is changing from positive to negative or vice-versa. This is also known as an inflection point. A slope may also occur where the gradient is constant, just a plain tilted at some angle. A slope is different from the step edge defined by Haralick (1982). Haralick classifies a pixel as a step edge if there is some point in the pixel's area having a zero-crossing of the second directional derivative taken in the direction of the gradient. The set of pixels called step edges has a non-empty intersection with the set we call slope, but neither set is completety contained in the other.

2.2.8 Summary of Classifier

A summary of the classification criteria can be found in Table 1. The first column in the table is the gradient magnitude and it must be non-negative since we are taking the positive square root. The next two columns represent the values of the extrema of the second directional derivative. The fourth and fifth columns represent the values of the first directional derivative in the direction of the extrema of the second directional derivative. The '+', '0', and '-' represent that the value is positive, zero, or negative, repectively. The '*' means it does not manner, it may take on any value. The final column is the classification the pixel is given if all the preceding conditions are met.

Pixel Classification Scheme

$\|\|\nabla f\|\|$	λ_1	λ_2	$\nabla f \cdot \omega^{(1)}$	$\nabla f \cdot \omega^{(2)}$	Label
0	-	-	0	0	Peak
0	-	0	0	0	Ridge
0	-	+	0	0	Saddle
0	0	0	0	0	Flat
0	+	-	0	0	Saddle
0	+	0	0	0	Ravine
0	+	+	0	0	Pit
+	-	-	-,+	-,+	Concave Hill
+	-	*	0	*	Ridge
+	*	-	*	0	Ridge
+	-	0	-,+	*	Concave Hill
+	-	+	-,+	-,+	Saddle Hill
+	0	0	*	*	Slope
+	+	-	-,+	-,+	Saddle Hill
+	+	0	-,+	*	Convex Hill
+	+	*	0	*	Ravine
+	*	+	*	0	Ravine
+	+	+	-,+	-,+	Convex Hill

Table 1

From the table, one can see that our classification scheme is complete. All possilble combinations of first and second directional derivatives have a corresponding entry in the table. Each pixel is given a unique label from the set of topographic labels.

3.0 Implementation

In this section we discuss the implementation of the classification scheme using the local facet model (Haralick,1981). It is important to note that this scheme is general enough that any method which can approximate the first and second order partials of the underlying digital image intensity surface at each sampled point may be used to implement the classifier.

3.1 Local Cubic Facet Model

In order to estimate the required partial derivatives we perform a least squares fit with a two dimensional surface, f, to a neighborhood of each pixel. It is required that the function f be continuous and have continuous first and second order partial derivatives with respect to r and c in a neighborhood around each pixel in the rc-plane.

We choose f to be a cubic polynomial in r and c expressed as a combination of discrete orthogonal polynomials. The function f is the best discrete least squares polynomial approximation to the image data in a neighborhood of each pixel. More details can be found in Haralick (1980). For a pixel with local coordinates (r,c) = (0,0), let

$$f(r,c) = k_1 + k_2 r + k_3 c + k_4 r^2 + k_5 rc + k_6 c^2 + k_7 r^3 + k_8 r^2 c + k_9 rc^2 + k_{10} c^3$$

It quickly follows that the needed partials evaluated at the center of the neighborhood, (0,0), are:

$$\partial f/\partial r = k_2$$

$$\partial f/\partial c = k_3$$

$$\partial^2 f/\partial r^2 = 2k_4$$

$$\partial^2 f/\partial c^2 = 2k_6$$

$$\partial^2 f/\partial r \partial c = k_5$$

Using the cubic facet we have:

$$\nabla f = (k_2, k_3)$$

$$\|\nabla f\| = (k_2^2 + k_3^2)^{1/2}$$

$$H = \begin{vmatrix} 2k_4 & k_5 \\ k_5 & 2k_6 \end{vmatrix}$$

Once the two eigenvalues and two eigenvectors of the Hessian are calculated, a pixel is classified by finding the appropriate entry in Table 1. Thresholds are used to determine if a value is negative, positive, or zero.

3.2 Summary of Implementation

In Section 3.2 we described a local, parallel process for pixel classfication which can be done in one pass through the image. At each pixel of the image the following three steps need to be performed:

(1) Calculate the coefficients of a two-dimensional cubic polynomial in an n-by-n neighborhood around the pixel. Only coefficients k_2 through k_6 are needed and they are calculated by convolving the appropriate masks over the image.
(2) Use the coefficients calculated in step 1 to find the gradient, gradient magnitude, and the eigenvalues and eigenvectors of the Hessian.
(3) Classify the pixel by finding the appropriate entry in Table 1.

4.0 Conclusions

In this paper we have given a precise mathematical description of the various topographical structures which occur in a digital image. Our complete set of topographical categories consists of {peak, pit, ridge, ravine, saddle, flat, slope, convex hill, concave hill, and saddle hill}. We have shown that a pixel may be classified based on its first and second order directional derivatives. The directional derivative classifier assigns a unique label to each pixel. We use a two-dimensional cubic polynomial fit based on the local facet model to estimate the directional derivatives of the underlying grey tone intensity surface. It was found that the calculation of the extrema of the second directional derivative can be done efficiently by forming the Hessian matrix and calculating its eigenvalues and their associated eigenvectors. The classification scheme was found to give very satisfying results on a number of test images.

Much further work needs to be done in this area. Different types of surface fitting need to be attempted, including polynomials of higher order. The fitting error needs to be taken into account when trying to decide into which class a pixel falls. Global knowledge of how these type of structures fit together could be used to undo the misclassification error due to noise. How the neighborhood size affects the surface fit and the classification scheme needs to be investigated. It is in these areas that future work will be directed.

5.0 References

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