A Statistical Framework for Geometric Tolerancing Manufactured Parts

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Abstract

An image is never noise free. Visual inspection of a part from its image is therefore affected by image errors. Understanding how image errors affect measurement precision is therefore critical for accurate inspection. In this paper, we lay out a statistical framework that allows to explicitly handle image errors and characterize their impact on measurement precision. A hierarchical model is also proposed to model manufacturing and measurement errors. Based on the model, a Bayesian technique is introduced to statistically infer the geometric tolerances of a manufactured part.

1 Introduction

Geometric tolerancing of machined parts requires first constructing tolerance zones from the data points and then inferring the geometric tolerances from the constructed tolerance zones. Since an image is never noise free, the noise associated with the data points affects the precision of the constructed tolerance zones, which, in turn, affects the precision of the tolerance measurements. Existing methods assume that the constructed tolerance zones are noise free. Image errors may arise from different sources. These errors collectively cause positional inaccuracies to image points. In this paper, we introduce a framework that allows to systematically propagate positional errors with image points to the constructed tolerance zones and then to the computed tolerance measurements. Given the uncertainty associated with each tolerance measurement, we present a Bayesian approach to statistically infer the true tolerance measurement.

2 Estimating Uncertainties of Tolerance Zones via Covariance Propagation

In this section, we first briefly introduce Haralick's covariance propagation theory and then show how to use it to estimate uncertainties with constructed tolerance zones. The tolerance zones considered here are composed of lines and circles. Understanding how image errors affect the parameters of lines and circles is important since the final tolerance measurements are derived from these parameters.

2.1 Covariance Propagation Theory

Haralick [1] recently proposed an analytic technique based on linearization for propagation of errors from input to output. Let $\hat{X}^{N\times 1} = (\hat{X}_1\ \hat{X}_2\ \ldots\ \hat{X}_N)^t$ be the observed input and $\hat{\Theta}^{K\times 1} = (\hat{\theta}_1\ \hat{\theta}_2\ \ldots\ \hat{\theta}_K)^t$ be the calculated output parameters. $\hat{\Theta}$ is determined by minimizing a scalar function $F(\hat{X}, \hat{\Theta})$. Then the input perturbation $\Sigma_{\Delta X}$ and the output perturbation $\Sigma_{\Delta\Theta}$ are related via

$$\begin{aligned}\Sigma_{\Delta\Theta} &= (\frac{\partial g(X,\Theta)}{\partial\Theta})^{-1}(\frac{\partial g(X,\Theta)}{\partial X})^t\Sigma_{\Delta X} \\ &\quad \frac{\partial g(X,\Theta)}{\partial X}((\frac{\partial g(X,\Theta)}{\partial\Theta})^t)^{-1}\end{aligned} \tag{1}$$

where $g(X,\Theta) = \frac{\partial F(X,\Theta)}{\partial\Theta}$ and X and Θ are the ideal input and output parameters.

2.2 Covariance Propagation for Least-Squares Line Fitting

Let $\hat{X} = (\hat{X}_1, \ldots, \hat{X}_N)$ be a vector of coordinates of image points. We want to fit a line with parameter $\Theta = (\theta, \rho)$ to points in $\hat{X}$. Assume $\hat{X}_n = (\hat{x}_n, \hat{y}_n)$, where $n = 1, \ldots, N$, results from perturbing ideal point $X_n = (x_n, y_n)$ lying on a line determined by Θ in the direction perpendicular to the fitted line. The scaler criterion function F that needs to be minimized in order to compute $\hat{\Theta}$ from $\hat{X}$ can be defined as

$$F(\hat{\Theta}, \hat{X}) = \sum_{n=1}^{N}(\hat{x}_n cos\hat{\theta} + \hat{y}_n \sin\hat{\theta} - \hat{\rho})^2$$

Plugging F into equation 1 leads to the covariance matrix of the estimated line parameters as

$$\sum_{\Delta\Theta} = \begin{pmatrix} \sigma_\theta^2 & \sigma_{\theta\rho} \\ \sigma_{\theta\rho} & \sigma_\rho^2 \end{pmatrix} = \sigma^2 \begin{pmatrix} \frac{1}{S_k^2} & \frac{\mu_k}{S_k^2} \\ \frac{\mu_k}{S_k^2} & \frac{1}{N} + \frac{\mu_k^2}{S_k^2} \end{pmatrix} \quad (2)$$

where σ^2 is the perturbation associated with each input point, and

$$\mu_k = \frac{1}{N}\sum_{n=1}^{N} k_n \quad S_k^2 = \sum_{n=1}^{N}(k_n - \mu_k)^2$$

and k_n is the distance between point (x_n, y_n) and a point on the line closest to the origin. Detailed derivations and geometric interpretation of the above equation may be found in [2].

2.3 Covariance Propagation in Least-Squares Circle Fitting

Let $\hat{X} = (\hat{X}_1, \ldots, \hat{X}_N)$ be a vector of image points that we want to fit a circle to. Let $\hat{X}_n = (\hat{x}_n, \hat{y}_n), n = 1 \ldots N$, be the observed points, the noisy instances of unperturbed points $X_n = (x_n, y_n)$. Assume $(\hat{x}_n, \hat{y}_n)$ are perturbed by iid Gaussian noise such that $\hat{X}_n$ is distributed as $N(X_n, \sigma^2 I)$.

Given a circle expressed by the equation

$$f(x, y, \Theta) = (x-a)^2 + (y-b)^2 - R^2 \quad (3)$$

the least-squares parameter estimate $\hat{\Theta} = (\hat{a}\ \hat{b}\ \hat{R})^t$ for $\Theta = (a\ b\ R)^t$ is obtained by minimizing

$$\varepsilon^2 = \sum_{n=1}^{M}(\sqrt{(\hat{x}_n - \hat{a})^2 + (\hat{y}_n - \hat{b})^2} - \hat{R})^2 \quad (4)$$

Substituting the above equation into equation 1 leads to

$$\Sigma_{\Delta\Theta} = 2\sigma^2[(\frac{\partial g}{\partial \Theta})^t]^{-1} \quad (5)$$

where

$$\frac{\partial g}{\partial \Theta} = \frac{2}{R^2}\sum_{n=1}^{N}\begin{pmatrix} s_n^2 & s_n t_n & R s_n \\ s_n t_n & t_n^2 & R t_n \\ R s_n & R t_n & R^2 \end{pmatrix} \quad (6)$$

where $s_n = (x_n - a)$ and $t_n = (y_n - b)$.

3 Impact of Image Errors on Tolerance Measurements

Assume we want to measure the straightness of an edge. According to ANSI standards [3], the tolerance zone for measuring the straightness of an edge consists of two parallel lines. Using the least-squares fitting method, we can fit a line to the edge points $(\hat{x}_n, \hat{y}_n)$, where $n = 1, \ldots, N$, yielding the best fitted line $(\hat{\theta}, \hat{\rho})$. Assume points $(\hat{x}_i, \hat{y}_i)$ and $(\hat{x}_j, \hat{y}_j)$ have the maximum and minimum distance to the best fitted line respectively, then the tolerance measurement $\hat{m}$ for the straightness is given by

$$\hat{m} = (\hat{x}_i - \hat{x}_j)\cos\hat{\theta} + (\hat{y}_i - \hat{y}_j)\sin\hat{\theta}$$

Linearizing the above equation around $\hat{\theta}$ yields

$$\sigma_m^2 = [(\hat{y}_i - \hat{y}_j)\cos\hat{\theta} - (\hat{x}_i - \hat{x}_j)\sin\hat{\theta}]^2\sigma_\theta^2 \quad (7)$$

where σ_θ^2 is the variance of the estimated line orientation parameter, which can be obtained from equation 2. σ_m^2 characterizes the precision of the straightness tolerance measurement due to image errors.

Assume again we want to measure the circularity or roundness of a machined circular feature. The tolerance zone for the measure of circularity of two dimensional data consists of two concentric circles, one representing the smallest circumscribing circle and the other representing the largest inscribed circle. The radius difference of the two circles measures the roundness of the circular feature. Using the least-squares fitting method, we can fit a circle to the edge points $(\hat{x}_n, \hat{y}_n)$, where $n = 1, \ldots, N$, yielding the best fitted circle $(\hat{a}, \hat{b}, \hat{R})$, where $(\hat{a}, \hat{b})$ and $\hat{R}$ are the center and radius of the best fitted circle. Assume points $(\hat{x}_i, \hat{y}_i)$ and $(\hat{x}_j, \hat{y}_j)$ have the maximum and minimum distance to the center of the best fitted circle respectively, then the circularity tolerance measurement $\hat{m}$ is

$$\hat{m} = \sqrt{(\hat{x}_i - \hat{a})^2 + (\hat{y}_i - \hat{b})^2} - \sqrt{(\hat{x}_j - \hat{a})^2 + (\hat{y}_j - \hat{b})^2} \quad (8)$$

Linearizing the above equation around $(\hat{a}, \hat{b})$ yields

$$\sigma_m^2 = (A \quad B)\Sigma_{(a,b)}(A \quad B)^t \quad (9)$$

where $A = \frac{\partial \hat{m}}{\partial a}$, $B = \frac{\partial \hat{m}}{\partial b}$, and $\Sigma_{(a,b)}$ is the covariance matrix of the center of the best-fitted circle. It can be obtained from 5. Here σ_m^2 characterizes the precision of the circularity measurement due to image errors.

4 Statistical Tolerance Inference

Given a part geometry, the vision inspection algorithm outputs a tolerance measurement $\hat{m}$. $\hat{m}$ contains two sources of errors: the manufacturing error ζ and the measurement error ξ. These two sources of errors, however, are indistinguishable from one another when the only information is from image. Let m be the unknown tolerance measurement of a part geometry in the absence of measurement errors. Here m only contains manufacturing error. Let μ

be the ideal tolerance measurement obtained in the absence of both measurement and manufacturing errors, then $\hat{m}$, m, and μ may be related by the following hierarchical model:

$$m = \mu + \zeta \qquad \hat{m} = m + \xi \tag{10}$$

where ζ and ξ are assumed to be independent Gaussian random variables with zero mean and variance of σ_t^2 and σ_m^2 respectively, where σ_m^2 can be obtained through error propagation as discussed in the last section and σ_t^2 may be estimated from the machine specification.

Let t be the required tolerance limit and $\hat{m}$ be a tolerance measurement from the vision system, we want to compute $P(m < t|\hat{m})$ i.e., the probability that the true tolerance measurement m is in-spec given the observed tolerance measurement $\hat{m}$.

Using Bayes theorem and hierarchical model in equation 10, we can show that the posterior probability of m given $\hat{m}$ follows a Gaussian distribution with mean of ν and variance of π^2, where

$$\nu = \frac{\mu\sigma_m^2 + \hat{m}\sigma_t^2}{\sigma_m^2 + \sigma_t^2} \qquad \pi^2 = \frac{\sigma_m^2\sigma_t^2}{\sigma_m^2 + \sigma_t^2}$$

i.e, $m|\hat{m} \sim N(\nu, \pi^2)$. As a result, we have

$$P(m < t|\hat{m} < t) = \Phi(\frac{t-\nu}{\pi}) \tag{11}$$

where $\Phi(x)$ is the cumulative distribution of a standardized normal distribution at x.

Define α to be the probability required to declare a tolerance measurement in spec. and let m_α be such that $\Phi(m_\alpha) > \alpha$. From the Bayes theorem, we have $\frac{t-\nu}{\pi} > m_\alpha$ if $P(m < t|\hat{m}) > \alpha$. In other words, to ensure a measured tolerance is in-spec, $\nu < t - \pi m_\alpha$.

With this, the theoretical misdetection (α) and false alarm (β) rates can be defined as follows let $m_0 = t - \pi m_\alpha$, then we have

$$\begin{aligned}\alpha &= P(\nu < m_0|m > t) \\ &= \frac{P(\nu < m_0)[1 - P(m < t|\nu < m_0)]}{1 - P(m < t)}\end{aligned}$$

where

$$\begin{aligned}P(m < t) &= \Phi(\frac{t-\mu}{\sigma_t}) \\ P(m < t|\nu < m_0) &= \int_{\nu=0}^{m_0} \Phi(\frac{t-\nu}{\pi})d\nu \\ P(\nu < m_0) &= \Phi(\frac{m_0 - \mu}{\frac{\sigma_t^2}{\sqrt{\sigma_m^2+\sigma_t^2}}})\end{aligned}$$

Similarly, the false alarm rate can be computed as

$$\begin{aligned}\beta &= P(\nu > m_0|m < t) = 1 - P(\nu < m_0|m < t) \\ &= 1 - \int_{m=0}^{t} \Phi(\frac{m_0 - C - Dm}{C\sigma_m})dm \end{aligned} \tag{12}$$

where $C = \frac{\mu\sigma_m^2}{\sigma_m^2+\sigma_t^2}$ and $D = \frac{\sigma_t^2}{\sigma_m^2+\sigma_t^2}$

5 Conclusions

In this paper, we described a framework that allows explicitly handle positional errors with image pixels and characterize their impact on precision of tolerance measurements. Also presented is a hierarchical model for modeling manufacturing errors and measurement errors. Based on the model, a Bayesian technique is introduced to statistically infer the geometric tolerances of a machined part and to theoretically compute the misdetection and false alarm rates of the inspection system.

References

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