ON SOME QUICKLY COMPUTABLE FEATURES FOR TEXTURE

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Abstract. - A set of features for the extraction of image texture is defined. The number of operations required to compute these features is proportional to the number of resolution cells on the image. Preliminary work has indicated that these features are helpful in the automatic identification of land use categories from aerial photography and photomicrographs of reservoir sandstones. In some cases identification accuracy near 90% has been obtained. However, because of small sample size limitations, further work must be done to verify these results.

The paper is divided into three parts. In the first part we discuss image normalization (a preprocessing function). In the second part we define the textural features and in the third part we illustrate the application of these features on image data.

I. Image Normalization. - The data which the sensors or instruments produce are not always in the kind of normalized form with which it makes sense to work. For example, many sensors or measuring instruments produce relative measurements, i.e. the measurements are correct up to an additive or multiplicative constant. Despite calibration efforts, this is particularly true for the camera-film-digitizer system which produce the digital magnetic tape containing the digitized image. Variations in lighting, lens, film, developer, and digitizer all combine to produce a grey tone value which is an unknown but usually monotonic transformation of the "true" grey tone value. Under these conditions we would certainly want two images of the same scene, one image being a grey tone monotonic transformation of the other, to produce the same results from the pattern recognition process. It is easy to show that normalization by equal probability quantizing guarantees that images which are monotonic transformations of one another produce the same results. It should be realized that something is not gained for nothing. The normalization is achieved by sacrificing the detailed grey information. After probability quantizing to 16 levels for example, an image which originally had 128 grey tones would only have 16 quantized grey tones and if equal probability quantizing were used, then the histogram of the quantized image would be uniform.

A precise statement of the effect of equal probability quantizing is as follows: Let X be a random variable with continuous cumulative probability function F_X. Let Q_1, the K level equal probability quantizing function for X be defined by $Q_1(x) = k$ if and only if

$$\text{lub}\left\{w \mid F_X(w) = \frac{k-1}{K}\right\} \leq x < \text{lub}\left\{w \mid F_X(w) = \frac{k}{K}\right\}.$$

For any strictly monotonic function g, define the random variable Y by $Y = g(X)$. Let Q_2, the K level equal probability quantizing function for Y, be defined by $Q_2(y) = k$ if and only if

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$$\mathrm{lub}\left\{w \mid F_Y(w) = \frac{k-1}{K}\right\} \leq y < \mathrm{lub}\left\{w \mid F_Y(w) = \frac{k}{K}\right\}.$$

The following lemma states that the equal probability quantization of X produces a random variable which is identical to the equal probability quantization of Y.

Lemma: $Q_1(X) = Q_2(Y)$.

Although the probability quantizing concept is simple, the application of it to digital image can easily give rise to problems since the cumulative probability function for a digitized image is not continuous. This implies that it is not usually possible to define a quantizing function which will make the grey tone histogram of the quantized image uniform. The problem is how to obtain a quantizing function which makes the grey tone histogram of the quantized image approximately uniform. We have had reasonable success with the following algorithm.

Let X be a non-negative random variable with cumulative probability function F_X. Let Q, the K level equal probability quantizing function for X, be defined by $Q(x) = k$ if and only if $q_{k-1} \leq x < q_k$. We define $q_0, q_1, q_2, \ldots, q_k$ in an iterative manner. Let $q_0 = 0$. Suppose q_{k-1} has been defined. Then let q_k be the smallest number such that

$$\left| \frac{1-F_X(q_{k-1})}{K-k+1} + F_X(q_{k-1}) - F_X(q_k) \right| \leq \left| \frac{1-F_X(q_{k-1})}{K-k+1} + F_X(q_{k-1}) - F(q) \right| \text{ for all real q.}$$

Figure 1 illustrates the equal probability quantizing algorithm.

II. Definition of Textural Features. - Other than some work with the Fourier, Hadamard transforms and the autocorrelation function, there exists little or no theory to aid in establishing what textural features should consist of. Rather the feature extraction operation is determined intuitively, rationalized heuristically and justified later pragmatically and empirically.

Let $L_x = \{1, 2, \ldots, N_x\}$ and $L_y = \{1, 2, \ldots, N_y\}$ be the x and y spatial domains and $L_y \times L_x$ be the set of resolution cells. Let $G = \{1, \ldots, N_g\}$ be the set of possible grey tones. Then a digital image I is a function which assigns some grey tone to each and every resolution cell; $I: L_y \times L_x \rightarrow G$. *

An essential component of our conceptual framework of texture are four closely related matrices from which all of our texture-context features are derived. These matrices are termed angular nearest neighbor grey tone spatial dependence matrices.

We assume that the texture-context information in an image I is contained in the over-all or "average" spatial relationship which the grey tones in image I have to one another. More specifically, we shall assume that this texture-context information is adequately specified by the matrix of relative frequencies P_{ij} with which two neighboring resolution cells separated by distance d occur on the image, one with grey tone i and the other with grey tone j (see Figure 2). Such matrices of spatial grey tone dependence frequencies are a function of the angular relationship between the neighboring resolution cells as well as a function of the distance between them. For angles quantized to 45° intervals the unnormalized frequencies are defined by:

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*The spatial domain $L_y \times L_x$ consists of ordered pairs whose components are row and column respectively. This convention conforms with the usual two subscript row-column designation used in FORTRAN.

$$P(i,j,d,0^{o}) = \#\{((k,l),(m,n)) \in (L_y \times L_x)\times(L_y \times L_x) \mid k-m=0,\ |l-n|=d,\ I(k,l)=i,\ I(m,n)=j\}$$
$$P(i,j,d,45^{o}) = \#\{((k,l),(m,n)) \in (L_y \times L_x)\times(L_y \times L_x) \mid (k-m=d,\ l-n=-d) \text{ or } (k-m=-d,\ l-n=d),\ I(k,l)=i,\ I(m,n)=j\}$$
$$P(i,j,d,90^{o}) = \#\{((k,l),(m,n)) \in (L_y \times L_x)\times(L_y \times L_x) \mid k-m=d,\ l-n=0,\ I(k,l)=i,\ I(m,n)=j\}$$
$$P(i,j,d,135^{o}) = \#\{((k,l),(m,n)) \in (L_y \times L_x)\times(L_y \times L_x) \mid k-m=d,\ l-n=d) \text{ or } (k-m=-d,\ l-n=-d),\ I(k,l)=i,\ I(m,n)=j\}$$

Note that these matrices are symmetric; $P(i,j;d,a) = P(j,i;d,a)$. The distance metric ρ implicit in the above equations can be explicitly defined by $\rho((k,l),(m,n)) = \max\{|k-m|, |l-n|\}$.

Consider Figure 3-a, which represents a 4 x 4 image with four grey tones, ranging from 0 to 3. Figure 3-b shows the general form of any grey tone spatial dependence matrix. For example, the element in the (2,1)-st position of the distance 1 horizontal P_H matrix is the total number of times two grey tones of value 2 and 1 occurred horizontally adjacent to each other. To determine this number, we count the number of pairs of resolution cells in R_H such that the first resolution cell of the pair has grey tone 2 and the second resolution cell of the pair has grey tone 1. In figures 3-c through 3-f we calculate all four distance 1 grey tone spatial dependence matrices. The appropriate frequency to probability normalization for these matrices can be easily computed.

From each of these four normalized angular nearest neighbor grey tone spatial dependence matrices at each distance we define the following textural features:

$$t_1 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i,j)^2$$

$$t_2 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i,j) \log P(i,j)$$

$$t_3 = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i-m)(j-m) P(i,j)}{s^2}, \text{ where } m = \sum_{i=1}^{N_g} i\, P(i,j), \quad s^2 = \sum_{i=1}^{N_g} (i-m)^2\, P(i,j)$$

$$t_4 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{1}{1+ i-j} P(i,j)$$

$$t_5 = \sum_{n=1-N_g}^{N_g-1} \left[\sum_{\substack{i-j \\ i-j=n}} P(i,j) \right]^2$$

Features t_1 and t_2 essentially measures homogeneity. The more homogeneous the character of the grey tone transition on the image, the larger t_1 will be and the smaller t_2 will be. The greater the heterogeneity, the smaller t_1 will be and the larger t_2 will be. Feature t_3 can be easily recognized as the correlation and it measures the linear dependence of the neighboring grey tones. Feature t_4 measures the extent to which the same or similar grey tones tend to be neighbors. Feature t_5 measures homogeneity of contrast, the extent to which grey tones of the same contrast tend to be neighbors.

Features of the same kind and of different angles can be used together by obtaining their average, range, or standard deviation. Information in the range or standard deviation tells about patterns having angular orientation preferences in the image. Small ranges or deviations indicate no angular preference; high ranges or deviations indicate strong angular preference.

III. Application of Textural Features. - The first application we illustrate is the use of textural features to help distinguish between photomicrographs of different kinds of reservoir rocks. The analysis of reservoir rock pore structure is important to geologists and petroleum engineers who are interested in obtaining a series of numerical descriptors of features which statistically describe porous media. These features are useful for the correlation and prediction of the physical properties of porous media including porosity, specific permeability and formation factor.

To explain the interpretation of some of the suggested textural features, let us consider the kinds of values they take on different kinds of porous rocks. Figure 4 shows two types of images and the values of textural features t_1 and t_3. The Dexter sandstone image has a smaller number of distinct grey tone transitions compared to the Upper Muddy sandstone image. In this respect, Dexter sandstone is more homogeneous than Upper Muddy sandstone. Hence the nearest neighbor grey tone spatial dependence matrix for the Dexter sandstone will have fewer entries of large magnitude and the nearest neighbor grey tone spatial dependence matrix for the Upper Muddy sandstone will have a large number of small entries. Feature t_1 has a smaller value for the Upper Muddy sandstone than for the Dexter sandstone.

The grain structure for the Dexture sandstone is more organized than the grain structure of the Upper Muddy sandstone. This organization implies that given the grey tone in any resolution cell, there is a higher probability of predicting (using a linear function) the grey tone in a neighboring resolution cell. This leads to a higher value for feature t_3.

In reference 1 an identification experiment was performed using 4 kinds of sandstone, the average and range of features t_1 and t_3 at distance 1 and the average and range of feature t_1 at distance 2. Linear discriminant functions were obtained using 84 training samples. A different test set of 60 samples was then processed. Over 88% of the test samples were correctly identified.

A second application we illustrate is the use of textural features to help distinguish between terrain land use categories on black and white aerial photography. This earth resource application is important in automatically making land use maps. Figure 5a illustrates a typical example of each one of fourteen land use categories: still water, heavily wooded, scrub, polluted water, marsh, turbulent water, single road, orchard, double road, swamp, railroad yard, residential without trees, urban, residential with trees. The digitized images are ordered according to their feature values t_2. Notice that as the imagery becomes more heterogeneous and complex, feature value t_2 gets larger. Figure 5b illustrates the ranges each of these categories takes for feature t_2. These ranges were obtained from a small set consisting of six samples for each category with the exception of still water which had twelve samples and residential with trees which had eight samples. Some of the categories look quite alike such as marsh and turbulent water or scrub and heavily wooded and other categories such as road or turbulent water had a wide range of appearance. In fact photointerpreters working from the digitized images could not do better than correctly identify 40% of the images. Working from the original aerial photographs they correctly identified 75% of the images. In reference 2 an automatic identification experiment was performed using 9 of these terrain categories and a decision rule which assumed the features were independent and uniformly distributed. The machine correctly identified

70% of the images using a procedure where the machine trained on 53 samples and was tested on the 54th sample.

Figure 6 illustrates the ranges for 2 other features. Notice that some categories which are not separable on the basis of t_2 are separable using some of the other features. Figures 5 and 6 clearly indicate the potential value which the textural features we defined can have in helping to discriminate between land use categories. They also indicate their limitations. In part these limitations are due to imperfections in the way the terrain land use category definitions are operationalized. This is the usual "ground truth" problem. In part the limitations are due to textural features which are not powerful enough. Hence, our future research will be directed along lines of clearing up these limitations.

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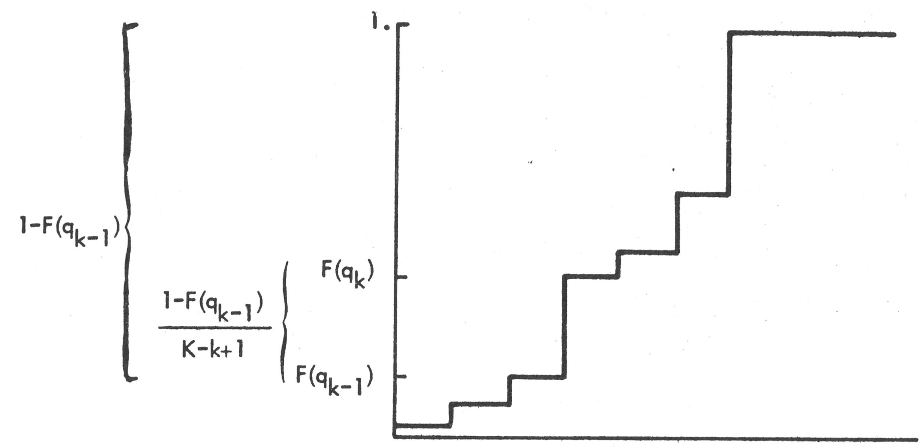


Figure 1. Illustrates the quantizing algorithm. At the kth iteration, $F(q_k\text{-}1)$ probability has already been allocated to k-1 levels and $1-F(q_k\text{-}1)$ probability remains to be allocated to K levels. If $1-F(q_k\text{-}1)$ probability is split up equally among the remaining K-k+1 quantizing level to be allocated, each level would get $\frac{1-F(q_{k-1})}{K-k+1}$. Since F is a step function, there is no guarantee that a q_k can be found and that $F(q_k)=F(q_{k-1}) + \frac{1-F(q_{k-1})}{K-k+1}$. Hence we look for a q_k which is closest to satisfying the equality.

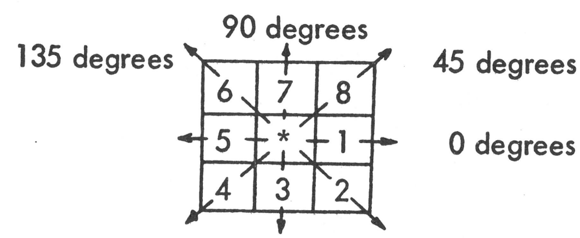


Figure 2. Resolution cells nos. 1 and 5 are the 0-degree (horizontal) nearest neighbors to resolution cell * , resolution cells nos. 2 and 6 are the 135-degree nearest neighbors, resolution cells 3 and 7 are the 90-degree nearest neighbors, and resolution cells 4 and 8 are the 45-degree nearest neighbors to * . (Note that this information is purely spatial, and has nothing to do with grey tone values).

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

Figure 3-a.

Grey Tone		0	1	2	3
Grey Tone	0	#(0,0)	#(0,1)	#(0,2)	#(0,3)
	1	#(1,0)	#(1,1)	#(1,2)	#(1,3)
	2	#(2,0)	#(2,1)	#(2,2)	#(2,3)
	3	#(3,0)	#(3,1)	#(3,2)	#(3,3)

Figure 3-b. This shows the general form of any grey tone spatial dependence matrix for an image with integer grey tone values 0 to 3. #(i, j) stands for number of times grey tones i and j have been neighbors.

0°

$$P_H = \begin{pmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Figure 3-c.

90°

$$P_V = \begin{pmatrix} 6 & 0 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

Figure 3-d.

135°

$$P_{LD} = \begin{pmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

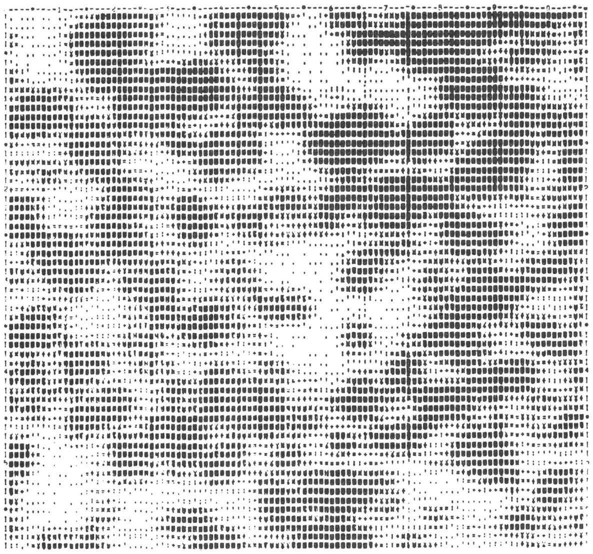
Figure 3-e.

45°

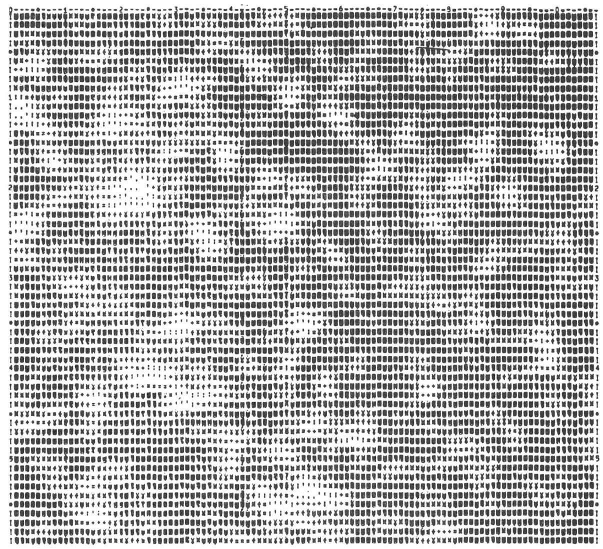
$$P_{RD} = \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Figure 3-f.

Figure 3. Illustrates simple example for the calculation of the nearest neighbor grey tone spatial dependence matrices.



Digitized Photomicrograph of
a. Dexter Sandstone



Digitized Photomicrograph of
b. Upper Muddy Sandstone

	a. t_1	a. t_3	b. t_1	b. t_3
0^o	0.0182	0.8093	0.0068	0.6532
45^o	0.0148	0.7098	0.0054	0.4657
90^o	0.0212	0.8522	0.0068	0.6463
135^o	0.0144	0.7031	0.0053	0.4247

Figure 4. Compares some typical feature values for Dexter sandstone and Upper Muddy sandstone.

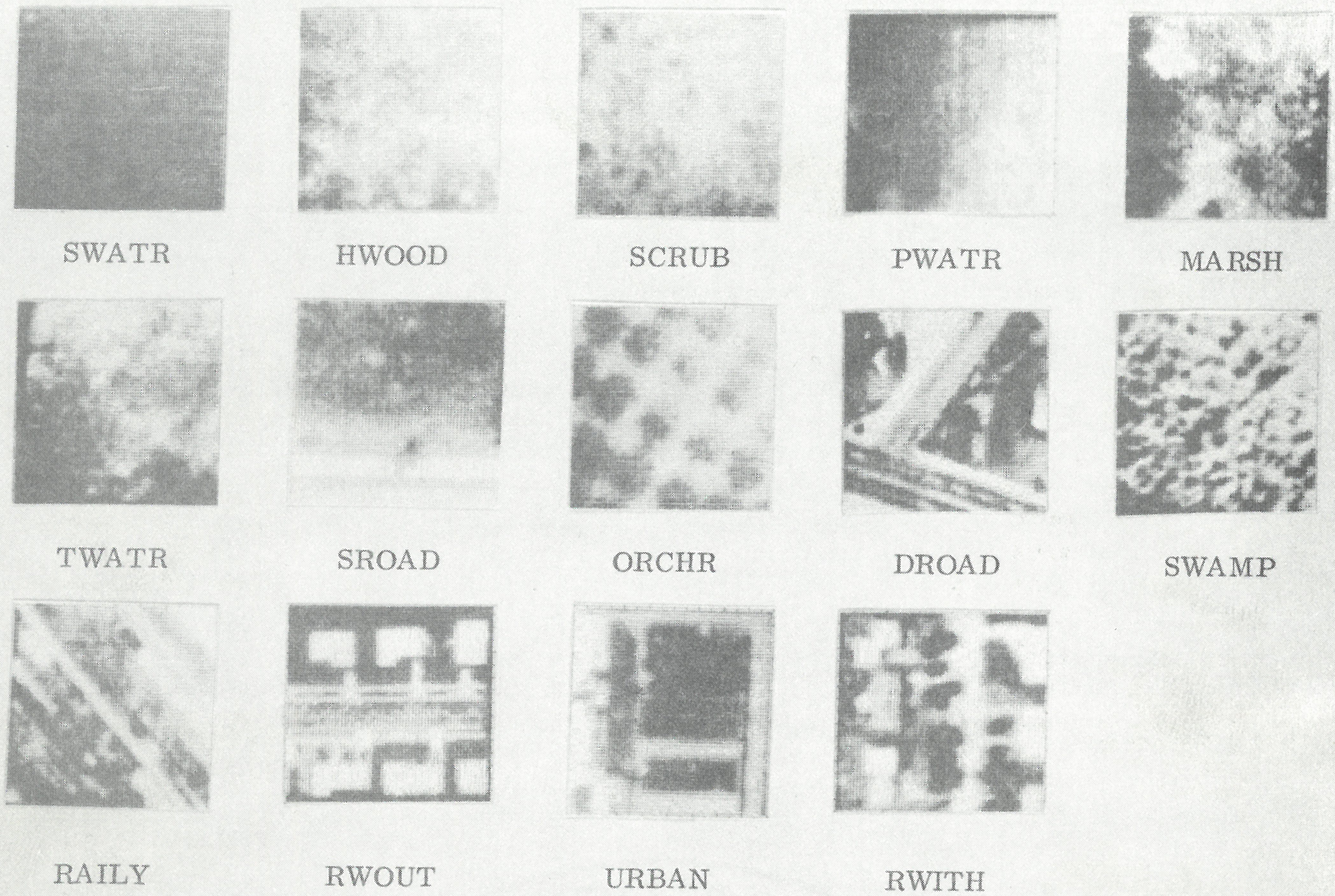


Figure 5a. Illustrates a typical example of the digitized images of fourteen land use categories.

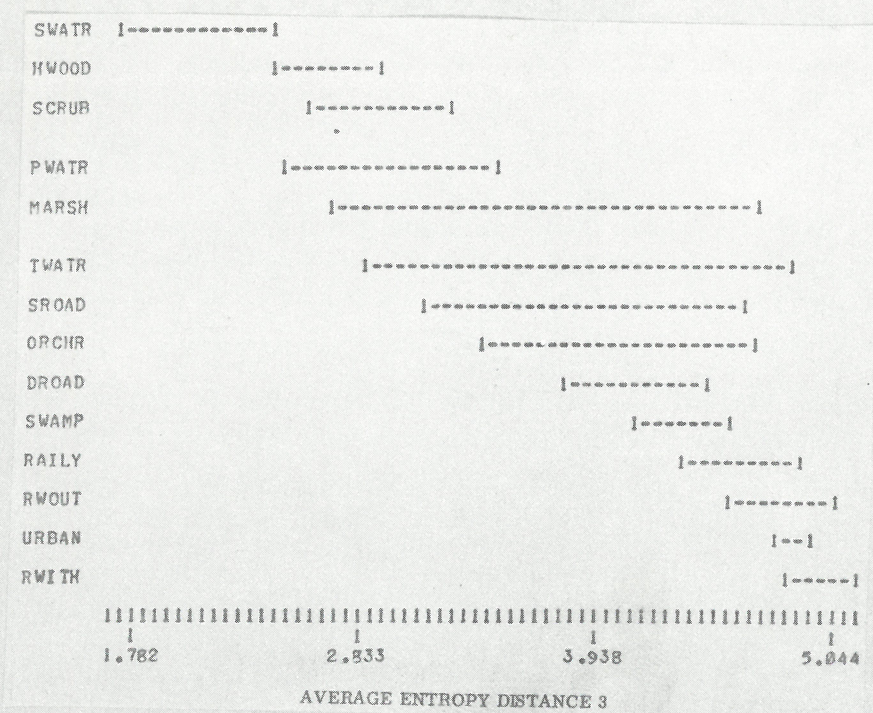
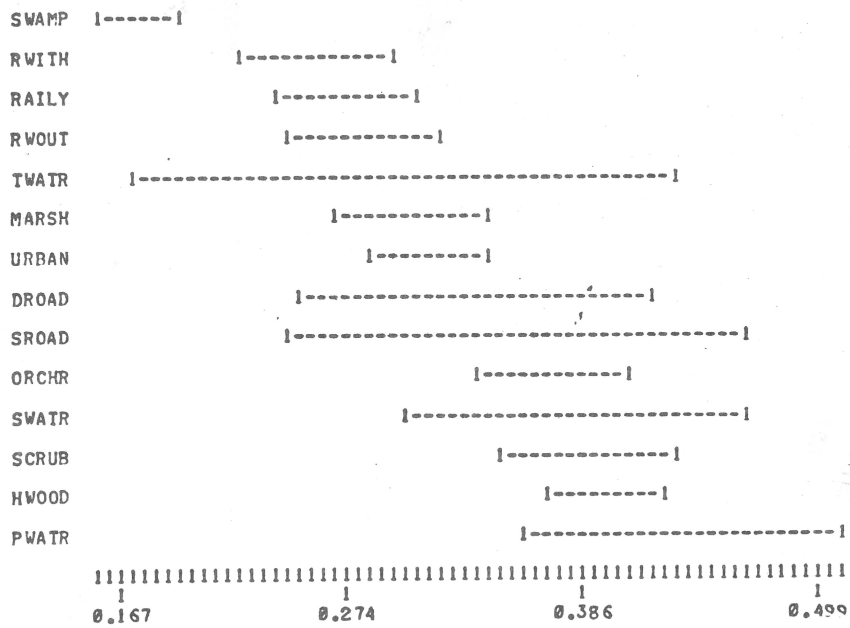
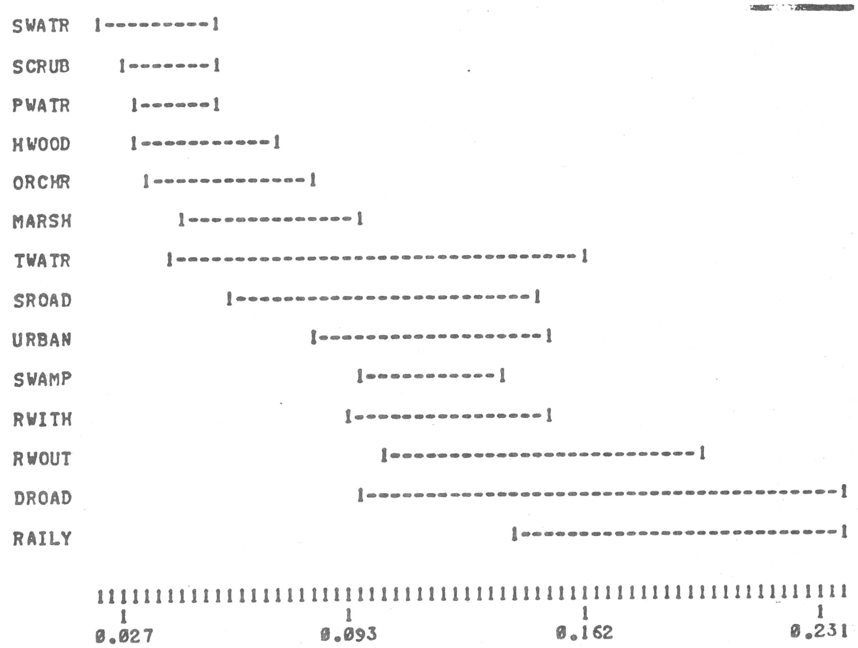


Figure 5b. Illustrates the ranges each of these categories takes for feature t_2.



AVERAGE CONTRAST DISTANCE 1



RANGE OF FEATURE 4 DISTANCE 1

Figure 6. Illustrates the ranges for the average of feature t_5 and the range of t_4 for each of the fourteen land use categories.