COMPARING THE LAPLACIAN ZERO CROSSING EDGE DETECTOR WITH THE SECOND DIRECTIONAL DERIVATIVE EDGE DETECTOR

Robert M. Haralick

Machine Vision International
Ann Arbor, MI 48104

Abstract

We present evidence that the Laplacian Zero-Crossing operator does not use neighborhood information as effectively as the second directional derivative edge operator. We show that the use of a Gaussian smoother with standard deviation 5.0 for the Laplacian of Gaussian edge operator with a neighborhood size of 50x50 both misses and misplaces edges on an aerial image of a mobile home park. Our results of the Laplacian edge detector on a noisy test checkerboard image are also not as good as the second directional derivative edge operator. We conclude by discussing a number of open issues on edge operator evaluation.

I. Introduction

Marr and Hildreth (1980) suggest that an edge detector first eliminates noise on the input image by smoothing with a sufficiently broad Gaussian filter, takes the Laplacian of the smoothed image, and marks pixels as edges if in some direction, the pixel on the convolved image has a zero crossing with high enough slope. In actual implementations, the Gaussian filtering and Laplacian operation are done with one Mexican hat filter whose kernel is the Laplacian of the Gaussian.

We tested the following zero crossing Laplacian edge detector. It uses a Gaussian smoother with standard deviation of 5 with a neighborhood size of 50x50, any pixel which had a slope greater than 10 zero-crossing of the smoothed Laplacian was assigned an edge. True edges were declared for any pixel of the no noise checkerboard which was black but bordered a white pixel or which was a white pixel and bordered a black pixel. Our results indicate that for a checkerboard made up of 20x20 checks and a contrast to noise ratio of 2:1, given a pixel is a true edge, the probability that the pixel is assigned an edge is .7217. Given that a pixel is assigned as edge, the probability that it is a true edge pixel is .7155.

We tried a directional derivative zero crossing example (Haralick, 1984) where we presmoothed with a Gaussian filter having a standard deviation of .88 followed by a 9x9 equally weighted fit to compute the facet coefficient. Figure 1 shows the checkerboard test image, the perfect edge image, the zero crossings of Laplacian of Gaussian with 5.0 standard deviation and the zero crossings of the second directional derivative edge detector (lower right). Here, .8391 is the probability of a pixel being a true edge pixel given that it is assigned an edge pixel. The probability of a pixel being assigned an edge given that it is a true edge is also equal to .8391. The directional derivative operator is better.

The edge operator evaluation situation is more complicated than it appears on the surface. From a signal content/noise content point of view, the standard deviation of the Gaussian filter must be set based on the size distribution of the homogeneous regions, their relative contrasts, and the amount of noise. A standard deviation of 5.0 for a Gaussian averager may leave objects such as the 20x20 checks intact, but would tend to smooth out of existence objects which are small or thin. Thus, there are circumstances in which a standard deviation of 5.0 would be inappropriately large and it is precisely for this reason when doing edge operator evaluation an upper window size limit must be selected to do the experiments rather than determine the largest window size which works well on the test image.

To see the folly of not fixing the upper limit size of the window consider an image whose size is as large as we like, whose left side is a noisy black and whose right side is noisy white. Suppose the signal to noise ratio is reasonable. Under these circumstances consider how we would want to evaluate edge operators. Since the geometry is utterly simple and the objects are as large as we would like, each edge operator proponent could find a window of sufficiently large size so that the edge operator produces a result of prespecified accuracy. Obviously, in this situation the above evaluation is meaningless. What we must do is perform the evaluation under conditions in which the pixel information provided to the edge operator is limited and then perform the evaluation under the limiting information conditions. Under these circumstances and edge operator could be said to be uniformly better than other edge operators if under each possible information limiting condition it performs better than all the other edge operators. Thus performance in controlled experiments must be performance as a function of information utilized. The key issue is how well does the operator utilize information in a bonded set.

II. Experiments

To show the problem of an excessively large standard deviation for the Gaussian smoother, we try to determine the edges of the aerial image of a mobile home park shown in figure 2. We perform three experiments. In the first experiment a Gaussian standard deviation of 5.0 is used with an adequate 45 by 45 window as the smoother preceding the Laplacian. The zero crossings obtained having a non-zero slope are shown in figure 3. Notice how many edges are not detected and that many edges are misplaced around nearly straight boundaries as well as around corners. This is only a reasonable edge image if the rows of the mobile homes were the desired objects. It is not a reasonable edge image if the boundary of the individual homes are desired.

In the second experiment a Gaussian standard deviation of .8 is used with an adequate 7x7 window as the smoother preceding the Laplacian. The zero crossings obtained having a slope greater than 2 are shown in figure 4. Twenty five percent of the pixels are assigned edges. Although noisy, at least this image shows the individual edges around the mobile homes.

The third experiment uses the second directional derivative zero-crossing edge operator. The equal weighted least squares bivariate cubic fit is done in a 7x7 neighborhood and a pixel is declared as an edge pixel if in the gradient direction a negatively sloped zero crossing of the second directional derivative occurs within a distance of .85 of the center of the pixel and the gradient magnitude is greater than 12. The resulting image has twenty five percent of the pixels assigned as edge and is shown in figure 5. The results are not as noisy as the Laplacian of figure 4. The edges are placed accurately and they tend to be connected.

We tried an interesting variation in which we used the fitting coefficients from the bivariate cubic fit to estimate the Laplacian. The resulting zero-crossings are shown in figure 6 in which the zero crossing threshold is chosen so that twenty five percent of the pixels are assigned as edges. They appear more connected that the zero crossings of the Laplacian of Gaussian operator.

III. Discussion

There are some interesting issues which have not yet been fully discussed or understood. Whether the edge operator be a Laplacian zero crossing one or a second directional derivative zero crossing one, the operator must estimate partial derivatives up through third order if zero crossing slope is used. For a fixed neighborhood size, what is the most effective way to estimate these partial derivatives? The Marr and Hildreth scheme is equivalent to averaging and then taking finite differences to compute the partial derivatives. The Haralich scheme performs a least squares estimate assuming a local cubic polynomial model. Finite differences and least squares yield the same result only when the polynomial model has as many parameters as pixels in the neighborhood. The least squares estimate can be generalized to a weighted least squares (Hashimoto and Sklansky (1983) have already suggested a binomial weighted least square) and it is possible to presmooth followed by a least squares estimate. It is also possible to pose the estimation problem as a robust estimation problem which in effect makes the weights used in the least squares fit adaptive.

The Marr Hildreth scheme chooses a direction which maximizes the zero crossing slope of the Laplacian. The Haralick (1984) and Canny (1983) scheme choose the gradient direction although they compute it in a different way. Are there other reasonable directional choices or computational techniques? What kind of experiment could be done to evaluate which is the better choice? What kind of analysis could be done to evaluate the choices in a theoretical way?

Both techniques cause edges to be displaced under certain conditions. In regions of non-linear gray tone intensity surface the Laplacian technique can spatially displace edges by as much as the standard deviation of the Gaussian smoother; it can even miss edges also (Berzins, 1984); (Leclerc and Zucker, 1984). Edges which curve rapidly around corners can be displaced by both techniques. There are difficulties around saddle points especially in the second directional derivative technique which requires a non-zero gradient.

These sorts of issues and problems need to be addressed. Perhaps this note will stimulate work in this area.

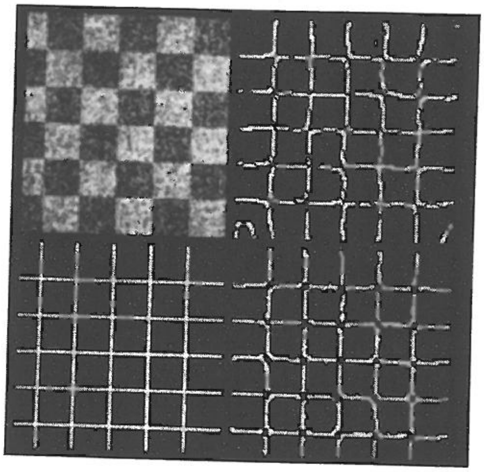

Figure 1 shows the checkerboard test image (upper left), the true edge image (lower left), the zero crossing of Laplacian image using a Gaussian standard deviation of 5.0 (upper right) and the second directional derivative edge operator with a Gaussian presmoother having standard deviation .88 followed by a equally weighted cubic fit in a 9x9 window (lower right).

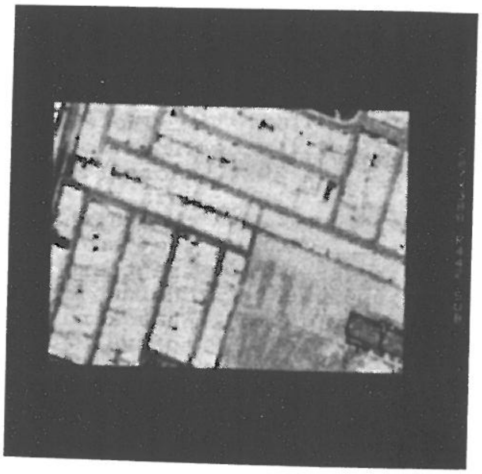

Figure 2 shows an aerial image of a trailer park

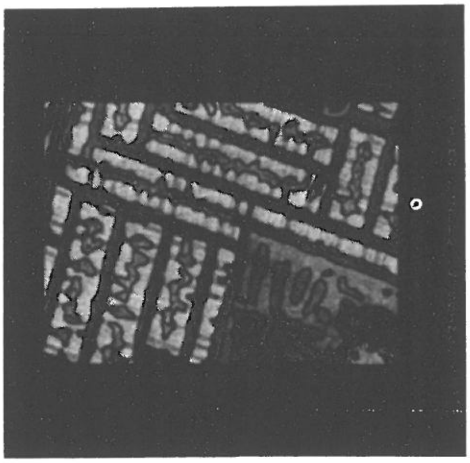

Figure 3 shows the zero crossings of a Laplacian edge detector having a Gaussian standard deviation of 5.0 and using a window of 45x45, 22% pixels are assigned as edges.

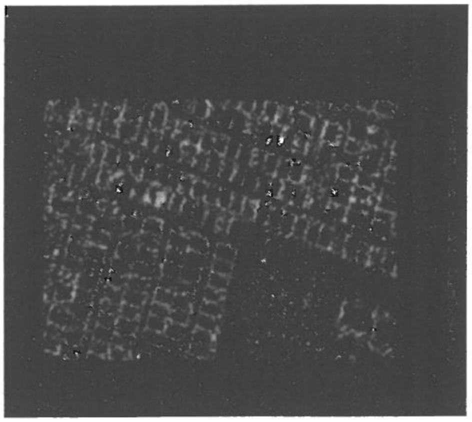

Figure 4 shows the zero crossings of a Laplacian edge detector having a Gaussian standard deviation of .8 and using a window of 7x7. 25% of the pixels are assigned as edges.

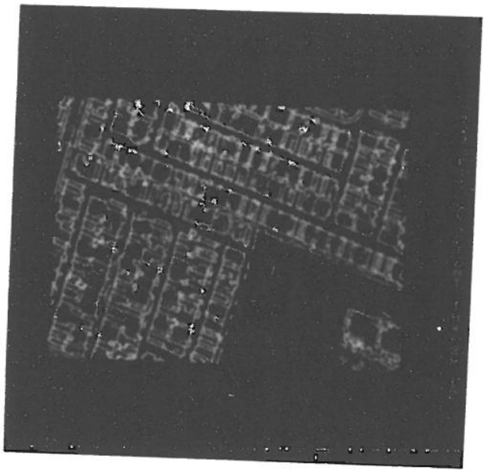

Figure 5 shows the second directional derivative edge detector using an equally weighted cubic fit in a 7x7 window. 25% of the pixels are assigned as edges.

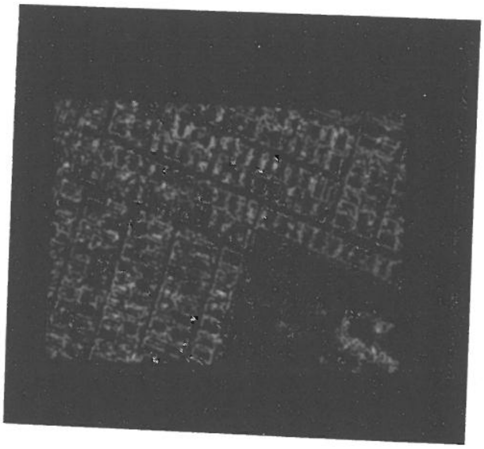

Figure 6 shows the zero crossings of a Laplacian edge detector using an equally weighted cubic fit in a 7x7 window. 25% of the pixels are assigned as edges.

References

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