THE TABLE LOOK-UP RULE

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Abstract

The table look-up rule problem can be described by the question: what is a good way for the table to represent the decision regions in the N-dimensional measurement space. This paper describes a quickly implementable table look-up rule based on Ashby's representation of sets in his constraint analysis. A decision region for category c in the N-dimensional measurement space is considered to be the intersection of the inverse projections of the decision regions determined for category c by Bayes rule in smaller dimensional projection spaces. Error bounds for this composite decision rule are derived: any entry in the confusion matrix for the composite decision rule is bounded above by the minimum of that entry taken over all the confusion matrices of the Bayes decision rules in the smaller dimensional project spaces.

Introduction

In the simple Bayes approach to pattern discrimination, a pattern measurement d is assigned to category c* only if

$$P(c^*|d)>P(c|d) \text{ for every } c \in C.$$

There are two distinct ways of implementing this assignment process: in the usual case, we take the pattern measurements to be vectors and for each category c, we estimate the conditional density $P(d|c)$ assuming a convenient multivariate form for $P(d|c)$. When a measurement d arrives for assignment, we plug it into the formula for $P(d|c)$ and assign it to category c* where

$$P(d|c^*)P(c^*)>P(d|c)P(c), \text{ for every } c \in C.$$

The only memory storage needed for this implementation process is for the parameters (mean and covariance) for each density. However, since a density must be computed each time a measurement needs to be assigned, the implementation tends to be compute-bound. This is a serious disadvantage for pattern discrimination using remotely sensed data because the number of measurements tends to be so high.

The other possible implementation procedure is to store the decision rule itself rather than the densities. Define $R(d)$ to be the category the decision rule assigns to measurement d. For a Bayes rule,

$$R(d) = c^* \text{ if and only if } P(c^*|d)>P(c|d) \text{ for every } c \in C.$$

Now when a measurement d arrives for assignment, we use d as an address to the table R and look-up the category assignment. When this method is implemented directly, memory storage is needed for the entire measurement space. This is a lot of memory especially when the dimension of measurement space gets to be above 4 or 5. Also a lot of computer processing time is needed to set up the table since the decision rule needs to be applied to each possible measurement to determine its assignment. However, since the category assignment is retrieved immediately by only an address calculation, the implementation tends to be fast. This is a clear advantage for pattern discrimination using remotely sensed data.

In this paper we explore the various ways a table look-up rule can be implemented and suggest a new implementation based on Ashby's technique of constraint analysis.

The Direct Table Look-Up Rule

Brooner, Haralick and Dinstein (1971) used a table look-up (discrete Bayes rule) approach on high altitude multiband photography flown over Imperial Valley, California to determine crop types. Their approach to the storage problem was to perform an equal probability quantizing from the original 64 digitized grey levels to ten quantized levels for each of the three bands: green, red, and near infrared. Then after the conditional probabilities were empirically estimated, they used a Bayes rule to assign a category to each of the 10^3 possible quantized vectors in the 3-dimensional measurement space. Those vectors which occurred too few times in the training set for any category were deferred assignment. Figure 1 illustrates the decision regions associated with such a table look-up discrete Bayes decision rule. Notice how the quantized multispectral measurement vectors can be used as an address in the 3-dimensional table to look-up the corresponding category assignment.

The rather direct approach employed by Brooner et al. has the disadvantage of requiring a rather small number of quantized levels. Furthermore, it cannot be used with measurement vectors of dimensions greater than four: for if the number of quantized levels is about 10, then the curse of dimensionality forces the number of possible quantized vectors to an unreasonably large size.

Recognizing the grey level precision restriction forced by the quantizing coarsening effect, Eppler, Helmke, and Evans (1971) suggest a way to maintain greater quantizing precision by defining a quantization rule for each category-measurement dimension as follows:

1. fix a category and a measurement dimension component;

2. determine the set of all measurement patterns which would be assigned by the decision rule to the fixed category;

3. examine all the measurement patterns in this set and determine the minimum and maximum grey levels for the fixed measurement component;

4. construct the quantizing rule for the fixed category and measurement dimension pair by dividing the range between the minimum and maximum grey levels into equal spaced quantizing intervals.

This multiple quantizing rule in effect determines for each category a rectangular parallelepiped in measurement space which contains all the measurement patterns assigned to it. Then as shown in Figure 2, the equal interval quantizing lays a grid over the rectangular parallelepiped. Notice how for a fixed number of quantizing levels, the use of multiple quantizing rules in each band allows greater grey level quantizing precision compared to the single quantization rule for each band.

A binary table for each category can be constructed by associating each entry of the table with one corresponding cell in the gridded rectangular parallelepiped. Then define the entry to be a binary 1 if the decision rule assigns a majority of the measurement patterns in the corresponding cell to the specified category; otherwise, define the entry to be a binary 0.

The binary tables are used in the implementation of the multiple quantization rule table look-up in the following way. Order the categories in some meaningful manner such as by prior probability. Quantize the multispectral measurement pattern using the quantization rule for category c_1. Use the quantized pattern as an address to look up the entry in the binary table for category c_1 to determine whether or not the pre-stored decision rule would assign the pattern to category c_1. If the decision rule makes the assignment to category c_1 the entry would be a binary 1 and, all is finished. If the decision rule does not make the assignment to category c_1, the entry would be a binary 0 and the process would repeat in a similar manner with the quantization rule and table for the next category.

Formally, this kind of table look can be described as follows. Let D be measurement space, the set of all possible N-tuple measurements. Let C be the set of categories. For each category $c \epsilon C$, let D_c be the quantized (discrete and finite) measurement space for category c. Let q_c be the quantizing rule for category c;

$$q_c: \ D \to D_c$$

Note that q_c could quantize some of the components of the N-tuple d to one possible value, in effect excluding that component from consideration.

Let T_c be the decision rule assignment for category c;

$$T_c: \ D_c \to \{0,1\}.$$

where

$$T_c(q_c(d)) = 1 \quad \text{if and only if } P(c|q_c(d)) > P(c'|q_{c'}(d)) \text{ for every } c' \epsilon C$$
$$= 0 \quad \text{otherwise.}$$

Then a measurement d is assigned to category c if $T_c(q_c(d)) = 1$.

One advantage to this form of the table look-up decision rule is the flexibility to use different subsets of bands for each category look-up table and thereby take full advantage of the feature selecting capability to define an optimal subset of bands to discriminate one category from all the others. A disadvantage to this form of the table look-up decision rule is the large amount of computational work required to determine the rectangular parallelepipeds for each category and the still large amount of memory storage required (about 5,000 8 bit bytes per category).

Shlien (1975) used a table look-up approach by storing in the table only the category assignments for measurement vectors which frequently occur. He used a hashing function to map the measurement vector into the table and reported that if the table is kept at no more than 75% full two distinct vectors are not likely to map to the same table address. Collisions were treated by using the independent double hashing technique described by Amble and Knuth (1974). Shlien indicated that most of the time about 6000 vectors in the table accounted for about 90% of the vectors occuring in an ERTS scene.

The Indirect Table Look-Up Rule

The limitation of the direct approach to the table look-up rule is memory storage. If only some assumptions could be made about the shape of the decision regions or some assumptions about the way a decision region can be represented, or some assumption about the form of the conditional probabilities; perhaps there could be some reduction in storage space associated with the table look-up rule.

Bledsoe and Browning (1959) suggested the following way to approximate the form of the joint probabilities without making a parametric assumption. Let M functions $h_1, \ldots, h_M$ be selected which map the N-dimensional measurement space to smaller K-dimensional discrete and finite feature spaces $F_1, \ldots, F_M$ respectively. Because of the discreteness and small dimensionality of feature space F_m, it is possible to store in tables all the joint probabilities $P_m(c,f)$ of a feature $f \epsilon F_m$ and a category $c \epsilon C$. To assign a category to a measurement d, the M features $h_1(d), \ldots, h_M(d)$ are determined and for each category c, the feature $h_m(d)$ is used as an address to retrieve the probability $P_m(c,h_m(d))$. Then an assignment is made to category c* only if

$$\prod_{m=1}^{M} P_m(c^*,h_m(d)) \geq \prod_{m=1}^{M} P_m(c,h_m(d)) \text{ for every } c \epsilon C.$$

This method is similar to the probability product approximations of Lewis (1959) and the more general product approximations of Ku and Kullback (1969).

Eppler (1974) discusses a boundary table look-up rule which enables memory storage to be reduced by five times and decision rule assignment time to be decreased by 2 times. Instead of pre-storing in tables a quantized measurement space image of the decision rule, he suggests a systematic way of storing in tables the boundaries or end-points for each region in measurement space satisfying a regularity condition and having all its measurement patterns assigned to the same category.

Let $D_q = D_1 \times D_2 \times \ldots \times D_N$ be quantized measurement space. A subset $R \subseteq D_1 \times D_2 \times \ldots \times D_N$ is a regular region if and only if there exists constants L_1 and H_1 and functions $L_2, L_3, \ldots, L_N, H_2, H_3, \ldots, H_N, L_n: D_1 \times D_2 \times \ldots \times D_{n-1} \to (-\infty,\infty)$, $H_n: D_1 \times D_2 \times \ldots \times D_{n-1} \to (-\infty,\infty)$ such that

$$\begin{aligned} R = \{(x_1,\ldots,x_N) \epsilon D | L_1 < x_1 < H_1 \\ L_2(x_1) < x_2 < H_2(x_1) \\ \vdots \\ L_N(x_1,x_2,\ldots,x_{N-1}) < x_N \\ < H_N(x_1,x_2,\ldots,x_{N-1})\} \end{aligned}$$

From the definition of a regular region, it is easy to see how the boundary table look-up decision rule can be implemented. Let $d = (d_1,\ldots,d_N)$ be the measurement pattern to be assigned a category. To determine if d lies within a regular region R associated with category c we look up the numbers L_1 and H_1 and test to see if d_1 lies between L_1 and H_1. If so, we look up the number $L_2(d_1)$ and $H_2(d_1)$ and so on. If all the tests are satisfied, the decision rule can assign measurement pattern d to category c. If one of the tests fails, tests for the regular region corresponding to the next category can be made.

The memory reduction in this kind of table look-up rule is achieved by only storing boundary or end-points of decision regions and the speed-up is achieved by having one-dimensional tables whose addresses are easier to compute than the three or more dimensional tables required by the direct table look-up decision rule. However, the price paid for these advantages is the regularity condition imposed on the decision regions for each category. This regularity condition is stronger than set connectedness but weaker than set convexity. (See Figure 3.)

Another approach to the table look-up rule can be based on Ashby's (1964) technique of constraint analysis. Ashby suggests representing in an approximate way subsets of Cartesian product sets by their projections on various smaller dimensional spaces. Thus, a subset of a Cartesian product set can be approximated by the larger set formed as the intersection of the inverse projections of the projections of the subset onto the smaller dimensional spaces. Using this idea for two-dimensional spaces we can formulate the following kind of table look-up rule.

Let $D_q = D_p \times D_2 \times\ldots\times D_N$ be quantized measurement space, C be the set of categories, and $J \subseteq \{1,2,\ldots,N\}^2$ be an index set for the selected two-dimensional spaces. Let the probability threshold α be given. Let $(i,j)\in J$; for each $(x_1,x_2)\in D_i \times D_j$ define the set $S_{ij}(x_1, x_2)$ of categories having the highest conditional probabilities given (x_1, x_2) by

$S_{ij}(x_1, x_2) = \{c\in C|P(c|x_1,x_2)>\alpha_{ij}\}$, where α_{ij} is the largest number which satisfies

$$\sum_{c\in S_{ij}(x_1,x_2)} P(c|x_1x_2)>\alpha$$

$S_{ij}(x_1,x_2)$ is the set of likely categories given that components i and j of the measurement pattern take the values (x_1, x_2).

The sets of S_{ij}, $(i,j)\in J$, can be represented in the computer by tables. In the $(i,j)^{th}$ table S_{ij} the $(x_1,x_2)^{th}$ entry contains the set of all categories of sufficiently high conditional probabilities given the marginal measurements (x_1,x_2) from measurement components i and j, respectively. This set of categories is easily represented by a one word table entry: a set containing categories c_1, c_7, c_9, and c_{12}, for example, would be represented by a word having bits 1, 7, 9, and 12 on and all other bits off.

The decision region R(c) containing the set of all measurement patterns to be assigned to category c can be defined from the S_{ij} sets by

$$R(c) = \{(d_1, d_2,\ldots, d_N)\in D_1 \times D_2 \times\ldots\times D_N | \{c\} = \bigcap_{(i,j)\in J} S_{ij}(d_i,d_j)\}$$

This kind of a table look-up rule can be implemented by using successive pairs of components (defined by the index set J) of the (quantized) measurement patterns as addresses in the just mentioned two-dimensional tables. The set intersection required by the definition of the decision region R(c) is implemented by takingthe Boolean AND of the words obtained from the table look-ups for the measurement to be assigned a category. Note that this Boolean operation makes full use of the natural parallel compute capability the computer has on bits of a word. If the k^{th} bit is the only bit which remains on in the resulting words, then the measurement pattern is assigned to category c_k. If there is more than one bit on or no bits are on, then the measurement pattern is deferred its assignment (reserved decision).

Thus we see that this form of a table look-up rule utilizes a set of "loose" Bayes rules in the lower dimensional projection spaces and intersects the resulting multiple category assignment sets to obtain a category assignment for the measurement pattern in the full measurement space.

Because of the natural effect which the category prior probabilities have on the category assignments produced by a Bayes rule it is possible for a measurement pattern to be the most probable pattern for one category yet be assigned by the Bayes rule to another category having much higher prior probability. This effect will be pronounced in the table look-up rule just described because the elimination of such a category assignment from the set of possible categories by one table look-up will completely eliminate it from consideration because of the Boolean AND or set intersection operation. However, by using an appropriate combination of maximum likelihood and Bayes rule, something can be done about this.

For any pair (i,j) of measurement components, fixed category c, and probability threshold β, we can construct the set of $T_{ij}(c)$ having the most probable pairs of measurement values from components i and j arising from category c. The set $T_{ij}(c)$ is defined by:

$$T_{ij}(c) = \{(x_1, x_2)\in D_i \times D_j|P(x_1, x_2|c)>\beta_{ij}(c)\},$$

where $\beta_{ij}(c)$ is the largest number which satisfies

$$\sum_{(x_1,x_2)\in T_{ij}(c)} P(x_1,x_2|c) \geq \beta$$

Tables which can be addressed by (quantized) measurement components can be constructed by combining the S_{ij} and T_{ij} sets. Define $Q_{ij}(x_1,x_2)$ by:

$$Q_{ij}(x_1,x_2) = \{c\in C|(x_1,x_2)\in T_{ij}(c)\} \cup S_{ij}(x_1,x_2)$$

The set $Q_{ij}(x_1,x_2)$ contains all the categories whose respective conditional probabilities given measurement values (x_1, x_2) of components i and j are sufficiently high (a Bayes rule criteria) as well as all those categories whose more probable measurement values for components i and j respectively are (x_1, x_2) (a maximum likelihood criteria). A decision region R(c) containing all the (quantized) measurement patterns can then be defined as before using the Q_{ij} sets:

$$R(c) = \Big\{ (d_1, d_2, \ldots, d_N) \epsilon D_1 \times D_2 \ldots \times D_N \Big| \{c\} = \bigcap_{(i,j)\, j} Q_{ij}(d_i, d_j) \Big\}$$

A majority vote version of this kind of table look-up rule can be defined by assigning a measurement to the category most frequently selected in the lower dimensional spaces.

$$R(c) = \Big\{ (d_1, d_2, \ldots, d_N) \epsilon D_1 \times D_2 \times \ldots \times D_N \Big| \# \{(i,j)\epsilon J | c \epsilon Q_{ij}(d_i,d_j)\} > \# \{(i,j)\epsilon J | c' \epsilon Q_{ij}(d_i,d_j) \text{ for every } c' \epsilon C - \{c\}\} \Big\}$$

The table look-up rule, as other kinds of rules, can also be used in a sequential decision tree procedure in the following way. Each level of the sequential procedure produces a tentative category assignment andthe tentative category assignments of level n become an additional dimension of measurement space for layer n+1. Hence measurement space grows by an added dimension each successive level. For all possible distinctions at each level, the sequential procedure is constrained to use the same feature set. Feature sets of different levels, however, can be different.

For each level a feature selection is performed on the measurement space defined for the level in order to determine the optimum measurement space dimensions. The selected measurement space dimensions are then used in a table look-up rule whose category assignments become an added dimension in measurement space for the next level.

Mathematically what happens is this. Let the level 1 decision rule f_1 be an ordinary table look-up rule. Suppose level 1,...level ℓ-1 decision rules $f_1, \ldots, f_{\ell-1}$ have already been defined. Define the level ℓ decision rule in an iterative way.

Let $N_{\ell-1}$ be the dimension of measurement space $D_{\ell-1}$ for the $(\ell-1)^{th}$ level. Define N_ℓ, the dimension of measurement space D_ℓ for the ℓ^{th} level by $N_\ell = N_{\ell-1} + 1$ and measurement space D_ℓ by

$$D_\ell = \{(d_1, \ldots d_{N_{\ell-1}}, d_{N_\ell}) | (d_1, \ldots, d_{N_{\ell-1}}) \epsilon D \text{ and}$$

$$d_{N_\ell} = f_{\ell-1}(d_1, \ldots, d_{N_{\ell-1}})\},$$

The feature selection procedure then uses the measurement space D_ℓ to produce the feature index set $J \subseteq \{1,\ldots,N\} \times \{1,\ldots,N\}$ as the index set which selects the features. Using a table look-up rule with J as described earlier, the decision rule f_ℓ is the determined.

4. Misidentification Error Bounds

Because the table look-rule based on tables in a smaller dimensional space than measurement space must necessarily give results which are less optimum than a Bayes rule, it is desirable to determine bounds on the misidentification error. To do this easily we will change our perspective slightly and think of the decision rule in the smaller dimensional space as its induced decision rule in the full measurement space. Ignoring for the moment the relationship between conditional probabilities and the decision rule definition, we will think of a decision rule as a partition in measurement space.

Suppose N different decision rules with no reserved decision regions are determined for measurement space D. Each decision rule can be characterized by the partition generated by its decision regions. Let $\{\Pi_{n1}, \Pi_{n2}, \ldots, \Pi_{nk}\}$ be the partition associated with the n^{th} decision rule. The cell Π_{nk} of the n^{th} partition is the region of all those measurements assigned by the n^{th} decision rule to the k^{th} category.

A composite decision rule (of the table look-up form) can be constructed from the N given decision rules in the following way: a measurement is assigned to the k^{th} category if each and every of the decision rules assigns it to the k^{th} category; if for any measurement a unanimous decision is not possible, then assignment for the measurement is reserved.

Let decision region Π_k of the composite decision rule be the set of all measurements assigned to the k^{th} category and decision region Π_o be the set of all reserved decisions. Then, by definition,

$$\Pi_k = \bigcap_{n=1}^{N} \Pi_{nk}, \quad k = 1, 2, \ldots, K$$

$$\Pi_o = D - \bigcup_{k=1}^{K} \Pi_k$$

Lemma 1 establishes: (1) upper bounds by category, for the probability of correct identification and the probability of misidentification; (2) lower bounds, by category, for the probability of reserving judgment and for the sum of misidentification probability and reserved judgment probability. The bounds are in terms of the correct identification and error rates in the confusion matrix for each of the N given decision rules.

Let $C = \{c^1, c^2, \ldots, c^K\}$ be the set of categories. Denote by $P_c(c^k)$ the probability of the composite decision rule correctly identifying a unit whose true category identification is c^k, by $P_e(c^k)$ the probability of the composite decision rule misidentifying a unit whose true category identification is c^k, and by $P_r(c^k)$ the probability of the composite decision rule reserving judgment on a unit whose true category identification is c^k. Denote by $P_c^n(c^k)$ the probability of the n^{th} decision rule correctly assigning a unit whose true category identification is c^k, and by $P_e^n(c^k,c^j)$ the probability of the n^{th} decision rule incorrectly assigning a unit whose true category identification is c^k to category c^j. The lemma states:

$$P_c(c^k) < \min_{\substack{n \\ n=1,\ldots,N}} P_c^n(c^k)$$

$$P_e(c^k) < \sum_{\substack{j=1 \\ j\neq k}}^{K} \min_{\substack{n \\ n=1,\ldots,N}} P_e^n(c^k,c^j)$$

$$P_e(c^k) + P_r(c^k) > \max_{\substack{n \\ n=1,\ldots,N}} \sum_{\substack{j=1 \\ j\neq k}}^{K} P_e^n(c^k,c^j)$$

$$P_r(c^k) > P(c^k) - \sum_{j=1}^{K} \min_{\substack{n \\ n=1,\ldots,N}} P_e^n(c^k,c^j)$$

It is also possible to use the error characteristics of a Bayes rule in the smaller dimensional spaces to determine error bounds on the bayes rule in full measurement space. Lemma 2 gives the upper bound

$$\sum_{d\in D} \min \{P(d,c^i),\ P(d,c^j)\}$$

for the probability $P_e(c^i{:}c^j)$ of a Bayes rule confusing categories c^i and c^j. Lemma 3 notes that when measurement space is transformed in any way by a mapping ϕ, then the upper bound of lemma 2 for the confusion error of the Bayes rule in the transformed space must increase. Lemma 4 states that if $\phi_1,\ldots,\phi_N$ are transformations of measurement space D to spaces $D_1,\ldots,D_N$ respectively, then the error bound of lemma 2 for the probability of a Bayes rule confusing category c^i and c^j itself can be bounded by

$$\min_{\substack{n \\ n=1,\ldots,N}} \sum_{d_n\in D_n} \min \{P(d_n c^i),\ P(d_n,c^j)\}$$

so that the total probability of error or a Bayes rule in measurement space D can be bounded by

$$\sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \min_{\substack{n \\ n=1,\ldots,N}} \sum_{d_n\in D_n} \min\{P(d_n,c^i),\ P(d_n,c^j)\}$$

The appendix gives statements of the lemmas without proof.

Appendix

The appendix gives precise statements of the lemmas. The proofs are omitted for brevity.

Lemma 1: Let $\{\Pi_{n1},\ldots,\Pi_{nk}\}$, $n=1,\ldots,N$ be given partitions of measurement space D. Define a new partition $\{\Pi_o,\Pi_1,\ldots,\Pi_K\}$ by

$$\Pi_k = \bigcap_{n=1}^{N} \Pi_{nk}$$

$$\Pi_o = D - \bigcup_{k=1}^{K} \Pi_k$$

Then, $P_c(c^k) < \min_{\substack{n \\ n=1,\ldots,N}} P_c^n(c^k)$

$$P_e(c^k) < \sum_{\substack{j=1 \\ j\neq k}}^{K} \min_{\substack{n \\ n=1,\ldots,N}} P_e^n(c^k,c^j)$$

$$P_e(c^k) + P_r(c^k) > \max_{\substack{n \\ n=1,\ldots,N}} \sum_{\substack{j=1 \\ j\neq k}}^{K} P_e^n(c^k,c^j)$$

$$P_r(c^k) > P(c^k) - \sum_{j=1}^{K} \min_{\substack{n \\ n=1,\ldots,N}} P_e^n(c^k,c^j)$$

Lemma 2: Let $P_e(c^i{:}c^j)$ be the probability that categories c^i and c^j are confused by a Bayes decision rule in measurement space D. Then,

$$P_e(c^i{:}c^j) < \sum_{d\in D} \min \{P(d,c^i),P(d,c^j)\}$$

Lemma 3: Let D be measurement space and $C = \{c^1,\ldots,c^K\}$ be the set of categories. Let a probability function P be given on DxC. Let a mapping $\phi{:}D\to D'$ be given which induces a probability function on D'. Then,

$$\sum_{d\in D} \min \{P(d,c^i),\ P(d,c^j)\} < \sum_{d'\in D'} \min \{P(d',c^i),\ P(d',c^j)\}$$

Lemma 4: Let D be measurement space and $C = \{c^1,\ldots,c^K\}$ be the set of categories. Let a probability function P be given on D x C. Let N mappings $\phi_n{:}D\to D_n$, $n=1,2,\ldots,N$ be given. Then an upper bound on the probability of error, P_e, for a Bayes rule in D can be given by:

$$P_e < \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \min_{\substack{n \\ n=1,\ldots,N}} \sum_{d_n\in D_n} \min \{P(d_n,c^i),\ P(d_n,c^j)\}$$

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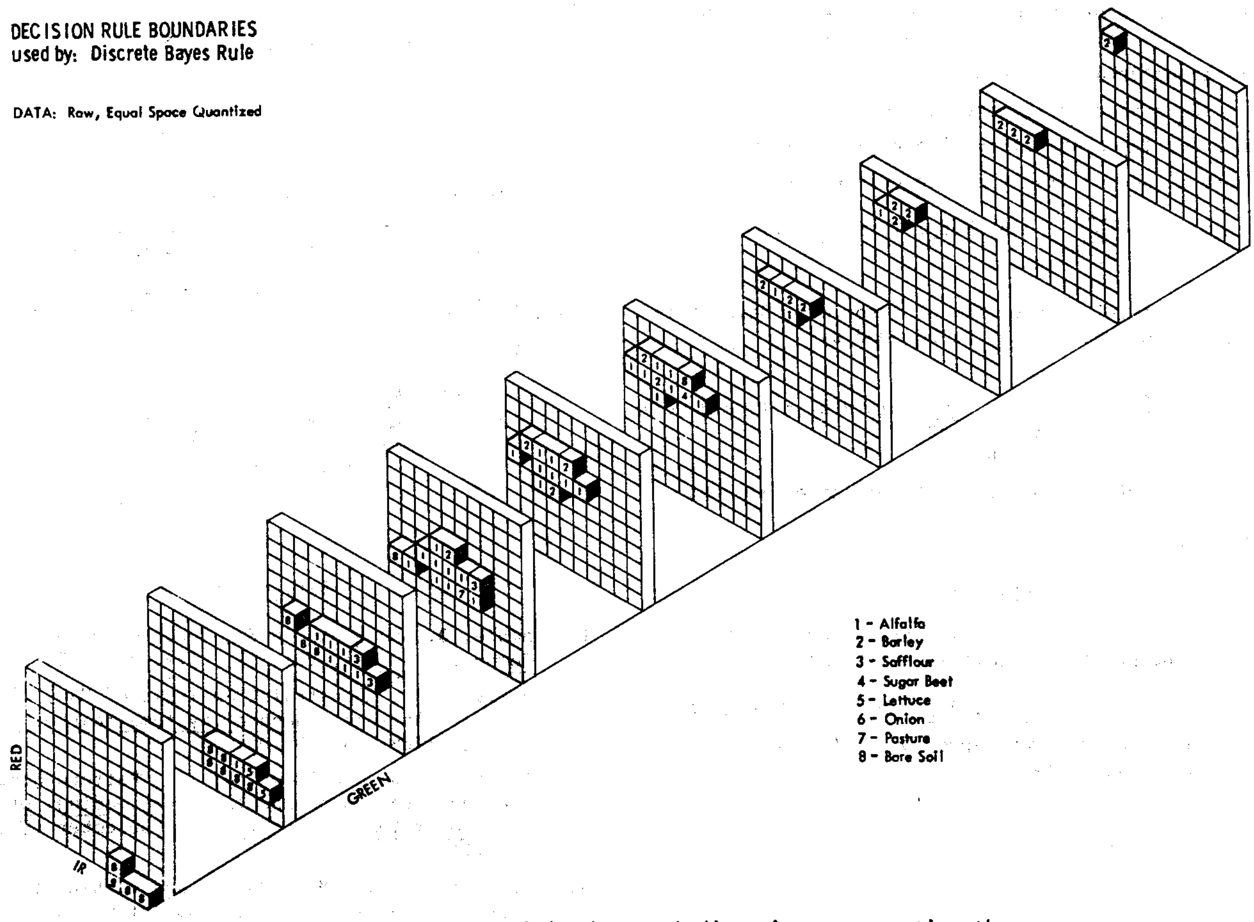


Figure 1. Viewed as an expanded cube, each dimension representing the spectral region of the three multiband images whose density values have been quantized to ten equally spaced levels, this sketch depicts the decision rule boundaries for each land-use category used by the discrete Bayes rule.

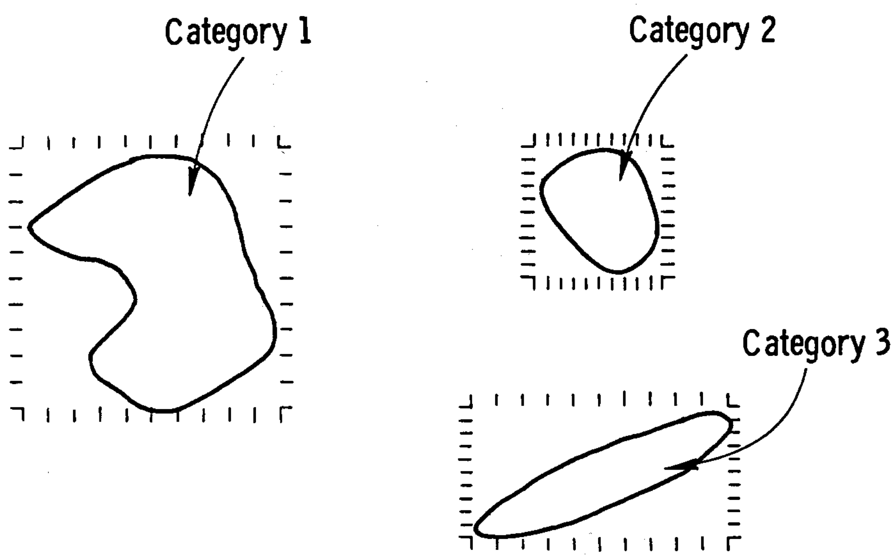
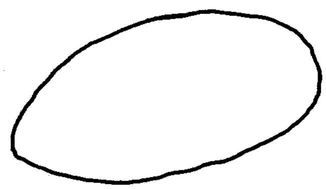
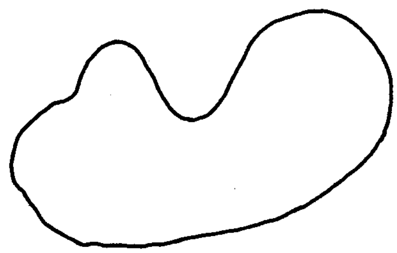


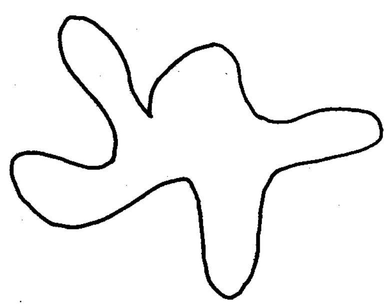
Figure 2 illustrates how quantizing can be done differently for each category thereby enabling more accurate classification by the following table look-up rule: (1) quantize the measurement by the quantizing rule for category one (2) use the quantized measurement as an address in a table and test if the entry is a binary one or binary zero, (3) if it is a binary one assign the measurement to category one; if it is a binary zero, repeat the procedure for category two.



Example showing that convex sets are regular



Example of a non-convex set which is regular



Example of a non-convex set which is not regular

Figure 3 illustrates the relationship between set convexity and regularity.