Subpixel Precision Corner Detection and Localization

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Abstrat:

One important class of interest points arises from location of sharp curvature in an extracted boundary sequence of row column positions. This paper establishes an optimal procedure for the segmentation of a boundary sequence into pieces which fit well to a straight line segments. Corners are localized as the intersection points of two successive fitted lines arising from the segmentation. The boundary sequence segmentation is optimal in the sense of creating maximal pieces for which the sum of the variances of the direction of the fitted lines arising from the pieces is minimal. Over one hundred thousand experiments were run to characterize the performance of the technique as a function of noise, number of points in a fitted line segment, and angle between successive line segments.

The Model

consider a situation in which points (x_i, y_i), $i = 1, \ldots, I$ are assumed to lie on an unknown straight line and the problem is to determine the parameters of the line. Then, we suppose (x_i, y_i) satisfies the model

$$\alpha x_i + \beta y_i + \gamma = 0 \quad i = 1, \ldots, I$$

where $\alpha^2 + \beta^2 = 1$. Now instead of assuming that the (x_i, y_i) are given, we assume that the noisy observations $(\hat{x}_i, \hat{y}_i)$ of (x_i, y_i) are given. Our model for $(\hat{x}_i, \hat{y}_i)$ is

$\hat{x}_i = x_i + \xi_i$; $\hat{y}_i = y_i + \eta_i$

where we assume that the random variables ξ_i and η_i are independent and identically distributed having mean 0, variance σ^2, and come from a distribution which is an even function. Hence,

$$E[\xi_i] = E[\eta_i] = 0$$

$$V[\xi_i] = V[\eta_i] = \sigma^2$$

$$E[\xi_i \xi_j] = \begin{cases} \sigma^2 & i = j \\ 0 & \text{otherwise} \end{cases}$$

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Now from the noisy observations $(\hat{x}_i, \hat{y}_i)$ we must estimate the parameters $\hat{\alpha}, \hat{\beta}$ of the unknown line. To do this we employ the principle of minimizing the squared residuals under the constraint, $\hat{\alpha}^2 + \hat{\beta}^2 = 1$

Letting

$$\hat{\mu}_x = \frac{1}{I}\sum_{i=1}^{I} \hat{x}_i$$

$$\hat{\mu}_y = \frac{1}{I}\sum_{i=1}^{I} \hat{y}_i$$

$$\hat{\sigma}_x^2 = \frac{1}{I-1}\sum_{i=1}^{I} (\hat{x}_i - \hat{\mu}_x)^2$$

$$\hat{\sigma}_y^2 = \frac{1}{I-1}\sum_{i=1}^{I} (\hat{y}_i - \hat{\mu}_y)^2$$

$$\hat{\sigma}_{xy} = \frac{1}{I-1}\sum_{i=1}^{I} (\hat{x} - \mu_x)(\hat{y}_i - \mu_y)$$

The minimization results in

$$\begin{pmatrix} \hat{\sigma}_x^2 & \hat{\sigma}_{xy} \\ \hat{\sigma}_{xy} & \hat{\sigma}_y^2 \end{pmatrix} \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = \lambda \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix}$$

$$\hat{\gamma} = -(\hat{\alpha}\hat{\mu}_x + \hat{\beta}\hat{\mu}_y)$$

Hence, the eigenvector $\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix}$ must be correspond to that eigenvalue $\hat{\lambda}$ of the sample covariance matrix having smallest eigenvalue. Therefore,

$$\hat{\lambda} = \frac{(\hat{\sigma}_x^2 + \hat{\sigma}_y^2) \pm \sqrt{(\hat{\sigma}_x^2 + \hat{\sigma}_y^2)^2 - 4(\hat{\sigma}_x^2\hat{\sigma}_y^2 - \hat{\sigma}_{xy}^2)}}{2}$$
$$= \frac{(\hat{\sigma}_x^2 + \hat{\sigma}_y^2) \pm \sqrt{(\hat{\sigma}_x^2 - \hat{\sigma}_y^2)^2 + 4\hat{\sigma}_{xy}^2}}{2}$$

The smaller eigenvalue corresponds to the minus sign. With $\hat{\lambda}$ determined, the corresponding unit length eigenvector can be determined.

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = \frac{1}{\sqrt{\hat{\sigma}_{xy}^2 + (\lambda - \hat{\sigma}_x^2)^2}} \begin{pmatrix} \hat{\sigma}_{xy} \\ \hat{\lambda} - \hat{\sigma}_x^2 \end{pmatrix} = \frac{1}{\sqrt{\hat{\sigma}_{xy}^2 + (\hat{\sigma}_y^2 - \lambda)^2}} \begin{pmatrix} \hat{\sigma}_y^2 - \hat{\lambda} \\ -\hat{\sigma}_{xy} \end{pmatrix}$$

Assuming that the variance of $\hat{\alpha}$ and $\hat{\beta}$ are small, and when $\sigma^2 \ll \sigma_x^2 + \sigma_y^2$,

$$E\left[(\hat{\theta} - \theta)^2\right] \leq \frac{\sigma^2}{(\sigma_x^2 + \sigma_y^2)(I-1)}$$

Segmantation Technique:

Let $S = (\hat{x}_n, \hat{y}_n, n = 1, \ldots, N)$ be a sequence of noisy observations of successive points on an unknown arc presumed to consist of a chain of line segments, where at least K successive observations occur for each line segment. One common intermediate level vision problem is to segment the sequence S into near maximal subsequences, each of which contain points lying on the same line segment. In the segmentation, the ending point of one subsequence can be the beginning point of the next subsequence. The common point is labeled a corner pont. The variance bound for $E[(\hat{\theta} - \theta)^2]$ suggests a way to create the segmentation.

We associate with each point $(\hat{x}_n, \hat{y}_n)$ of the sequence S a maximal sequence L_n of points prior to $(\hat{x}_n, \hat{y}_n)$ and a maximal sequence R_n of points after $(\hat{x}_n, \hat{y}_n)$. The sequences L_n and R_n are defined using the bound

$$\frac{\sigma^2}{(\sigma_x^2 + \sigma_y^2)(I-1)}$$

on the variance of the fitting angle. So long as successive points after $(\hat{x}_n, \hat{y}_n)$ are taken for the fitted line and the underlying model is valid, on each successive point is added to the fit the denominator must increase because $(\sigma_x^2 + \sigma_y^2)$ increases and $(I-1)$ increases. Should the bound increase instead of decrease, it must be due to the invalidity of the underlying model and this invalidity could only be caused by taking a successive point which is not on the underlying line. This observation motivates the following procedure for defining the maximal sequences R_n and L_n.

Let R_n^M be the sequence of M points following $(\hat{x}_n, \hat{y}_n)$ in the given sequence S. $R_n^M = ((\hat{x}_{n+1}, \hat{y}_{n+1}), \ldots, (\hat{x}_{n+m}, \hat{y}_{n+m}))$. Associated with the fit for the points of R_n^M are the calculated quantities $\hat{\sigma}_x^2$, $\hat{\sigma}_y^2$, and $\hat{\lambda}$. We estimate the variance bound $b_{R_n}^M$ for the angle of the fitted line of M points by $\hat{b}_{R_n}^M$ where

$$\hat{b}_{R_n}^M = \frac{\hat{\lambda}}{(\hat{\sigma}_x^2 + \hat{\sigma}_y^2 - 2\hat{\lambda})(M-1)}$$

We define the sequence R_n by $R_n = R_n^{M^*}$ where M^* is the largest index greater than K for which $\hat{b}_{R_n}^{M^*} < \hat{b}_{R_n}^{M^*+1}$. The sequence L_n is defined in an analogous way.

Associated with the fit of the points in L_n and R_n are the estimators $(\hat{\alpha}_{R_n}, \hat{\beta}_{R_n})$ and $(\hat{\alpha}_{L_n}, \hat{\beta}_{L_n})$ for the angle of the line. The change in angle at $(\hat{x}_n, \hat{y}_n)$ can be estimated by $\hat{\gamma}_n$ where

$$\cos \hat{\gamma}_n = \hat{\alpha}_{R_n} \hat{\alpha}_{L_n} + \hat{\beta}_{R_n} \hat{\beta}_{L_n}$$

A bound for the variance of $\hat{\gamma}_n$ is given by the sum of the variance bounds for angle arising from the left and right fits.

$$V[\hat{\gamma}_n] \leq \hat{b}_{L_n} + \hat{b}_{R_n}$$

This suggests that a test statistic suitable for measuring the significance of the angle change γ_n away from zero is

$$\hat{t}_n = \frac{\cos^{-1}(\hat{\alpha}_{R_n} \hat{\alpha}_{L_n} + \hat{\beta}_{R_n} \hat{\beta}_{L_n})}{\sqrt{\hat{b}_{L_n} + \hat{b}_{R_n}}}$$

Values of $\hat{t}_n$ which are large are suggestive that a corner exists at point $(\hat{x}_n, \hat{y}_n)$ having angle $\hat{\gamma}_n$.

The Algorithm:

The input to the corner detection algorithm is a sequence C of m (x,y) pairs. $C =< p_1, p_2, ... p_m >$, where each p_i represents an (x,y) point. These pairs of (x,y)'s represent pixel positions obtained from a digital curve and the order of the points in the sequence is the order in which they occur on the digital curve. The corner detection algorithm partitions C into a set of "digital straight" line segments and computes the corner points to subpixel accuracy as the intersection points of adjacent fitted lines resulting from the segmentation of C.

The algorithm partitions C in two steps as follows:
Step I (initial curve partition): For a given k, $(k = 5)$, define the subsequence $L(1,k)$ of C, $L(1,k) = < p_1, p_2, ..., p_k >$, as the first k points from the sequence C. Using a total least squares criterion, we fit a straight line, l to the subsequence, $L(1,k)$. Then we compute the function $EV(L(1,k))$, the estimated variance of the direction of the line l (section 3). We add the next point, p_{k+1}, to the sequence $L(1.k)$ to form a new sequence, $L(1,k+1)$ and compute $EV(L(1,k+1))$. As we have shown in section 3, the expected value of $EV(L(1.k+1))$ is less than or equal to $EV(L(1,k))$, as long as the new point, p_{k+1}, really belongs to same fitted line, l as the points in the old sequence $L(1,k)$.

The point adding process continues until at position $m, m > k$, we find a j-long sequence, $j \geq J(m;\hat{\sigma},\beta)$, of monotonically increasing variance estimates $EV(L(1,m)), EV(L(1,m+1)), ..., EV(L(1,m+j))$. That is, the last j values of the EV sequence is monotonically increasing. Then we mark p_m as a potential break point on the curve C.

The initial curve partition algorithm continues by making the point p_m be the first point in the next subsequence of C and locates the next potential break point in a similar manner to how the first break point was located. It continues in this manner until the end of C is reached. The number $J(m;\hat{\sigma},\beta)$ is determined from the experiments described in section 5.1.2 and satisfies the relation

$$Prob(j \geq J(m;\hat{\sigma},\beta) | \text{ noisy points arise from points on the same underlying line }) \leq \beta.$$

Step II (Iteration process): The purpose of this step is to recompute a new sequence of break points using the entire subsequence of points between each pair of potential break points from the previous iteration. The iteration process continues until the set of new break points is identical with the set of old break points. The iteration process is related to an iso-data like clustering with the spatial constraint of point order being given and with the fitted line direction variances being the quantities are reduced at each iteration.

At the beginning of each iteration, each of the potential break points is checked for its statistical significance. A potential break point will be eliminated if the difference between line directions for two successive fitted lines it separates is NOT statistically significant. Significance is determined with a t-test for the equality of line directions. A t value which is too small would suggest that there is no statistical difference between the directions of the two successive lines. Therefore, the break point should be eliminated from the curve C. (The details of the t-test will be discuss later.)

Let $Q_0 =< p^0_{j_1}, p^0_{j_2}, ... p^0_{j_m} >$, be the sequence of the potential break points on C that are marked by Step I. Let $Q_i =< p^i_{j_1}, p^i_{j_2}, ..., p^i_{j_n} >$, be the sequence of the statistically significant

potential break points determined at the i^{th} iteration. Note that the p_j's partition the input curve C into $n+1$ subsequences. Let $< s_1^i, s_2^i, ..., s_{n+1}^i >$ be these subsequences. We take the first two subsequences. $s_1 =< p_1^i, ..., p_{j_1}^i >$ and $s_2 =< p_{j_1}^i, ..., p_{j_2}^i >$ and compute the forward EV sequence starting from p_k^i to $p_{j_2}^i$ and the backward EV sequences starting from $p_{j_2-k}^i$ to p_1^i. To make our explanation simple, let u be the total length of the two sequences $s1$ and $s2$, and let $< EV(L(1,q)), q = k, ..., u >$, be the forward EV sequence and $< EV(L(u,q)), q = u-k, ..., 1 >$, be the backward EV sequence. To find the new position of the potential break point in $< p_1^i, ..., p_{j_1}^i, ..., p_{j_2}^i >$, the algorithm adds the corresponding pairs of variance estimates from the forward and the backward sequences and computes at each position r the sum $EV(L(1.r)) + EV(L(u,u-r))$. Let q be the position having the minimum summed variance estimates. If p_q is differ from p_{j_1}, then p_q becomes the revised first potential break point in Q_i. The next break point is determined with the subsequences s_2 and s_3. And so on.

The iterations continue until there is no change. If a potential break point becomes an actual break point, then since each iteration never increases the summed variances and since the variances are non-negative, the iterations must converge. If some potential break point does not become an actual break point because of merging, then the next iteration will have fewer break points. Since the number of break point is bounded below by zero, the iterations again must converge.

Let $< l_1, l_2, ..., l_m >$ be the least square fitted lines for each subsequence of the final partitioned C. The intersection point of each pair of two successive lines l_i and l_{i+1} are the estimated corner point positions. They are estimated to subpixel precision since the intersection point is not required to lie on a digital grid point.

The t-test

In the following, we will give the detail of how a potential break point is tested for its statistical significance.

Let s_i and s_{i+1} be any two successive point subsequences of C. Let the number of points in s_i be N_1 and the number of points in s_{i+1} be N_2. Let θ_1 denote the direction of the fitted line of s_i and θ_2 denote the direction of the fitted line of s_i. Then the t-statistic having $N_1 + N_2$ - 2 degrees of freedom is defined by

$$t_{N_1+N_2-2} = \frac{|\theta_1 - \theta_2|}{\sqrt{\frac{N_1 * EV(L_1) + N_2 * EV(L_2)}{N_1+N_2-2} * \frac{N_1+N_2}{N_1 * N_2}}}$$

Since the angle of the same line can be $\theta + n\pi$, for any integer n, for $0 \le \theta_1 \le \pi$ and $0 \le \theta_2 \le \pi$, $|\theta_1 - \theta_2|$ is computed as $min\{|\theta_1 - \theta_2|, |\theta_1 - \theta_2 - \pi|, |\theta_1 - \theta_2 + \pi|\}$

A value $T_{N_1+N_2-2,\alpha}$ can be determined which satisfies

$$Prob(t_{N_1+N_2-2} > T_{N_1+N_2-2,\alpha}) = \alpha.$$

When $t_{N_1+N_2-2} < T_{N_1+N_2-2,\alpha}$, the hypothesis of the equality of line direction is accepted. This situation casts doubt on the significance of the segmentation. Hence, the points of the sequence that the potential break point separates would be merged. When $t_{N_1+N_2-2} > T_{N_1+N_2-2,\alpha}$ the hypothesis of the equality of line direction is rejected. This situations provides evidence for the statistical significance of the segmentation. In this case, the potential break point becomes an actual break point.