Left Ventricle Longitudinal Axis Fitting and its Apex Estimation using a Robust Algorithm and its Performance: A Parametric Apex Model

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Abstract

For complete automatic left ventricle border detection in a cardiac frame, the apex needs to be located. As the apex zone has less contrast and is harder to identify [1], [4] in the gray scale left ventriculograms, we use the left ventricle's longitudinal axis [2] to assist in apex location. To *automatically* find the longitudinal axis of the left ventricle in any frame, we find the longest segment from either the anterior aspect of aortic valve or the inferior aspect of the aortic valve to the left ventricle border. We assume that the ruled surface generated by the sequence of longitudinal axes through the cardiac cycle is of sufficiently simple form, so that the perturbation error, especially the large errors, between the *automatically measured* axis and the *physician defined* axis can, in part, be filtered out by a robust procedure.
To *discriminate* those automatically determined axes that might differ significantly from the physician defined ground truth, we use Huber's weight function in an iterative reweighted least square robust fitting. We show that when the variance of the inlier noise is 1 mm^2 and variance of the outlier noise is 100 mm^2 and outliers occur in 15% of the longitudinal axes, then the robustly estimated axis end points have a root mean square error of 2.2 mm. By contrast, with only inlier noise and using only 85% of the longitudinal axis data, the ordinary least squares estimate has a root mean square error of 1.34 mm. With both inlier and outlier noise, an ordinary least squares estimate would produce estimated apex vertices having a root mean square error of 10.5 mm. We demonstrate that for 90% of 1200 frames of clinical data, the automatically determined apex location is less than an arc length distance of 1% of the ventricle border length from the ground truth apex location as delineated by the cardiologist.

Key Words: Longitudinal Axis (LA), Ruled Surface, Left ventricle (LV), Apex, Robust Estimation, Outliers

I. Introduction

The apex point is the farthest point along the LA from the aortic valve (AoV) plane. To keep track of the changing shape at the apex, we must keep track of the apex points of the LA. During the systole and diastole periods of the cardiac cycle, the heart's motion causes the LA to change its length and inclination (position). When these *automatically measured longitudinal axes* are observed frame by frame, they fall into two classes. Most measured axes are small perturbations from the axis that a physician would delineate. A few have very large perturbations and are called *outliers*. Because outliers have an unusually great influence on *least square estimators*, it would be inappropriate to use least squares estimation in such a situation. Therefore, we determine the apex points of the LA in each frame of the cardiac cycle by robustly estimating the ruled surface coefficients using an iterative reweighted least squares (IRLS) fit.

II. Formal Problem Statement

Given the noisy *perturbed automatically measured* LA data of the LV, we must robustly estimate the ruled surface and its coefficients using an iterative reweighted least squares. From the fitted surface, we can then produce fitted estimates of the *measured longitudinal axis*.
We first give the noise model for the measured apex data for any frame f and then express the same model in matrix form for the complete cardiac cycle. Let $[x(f), y(f)], f = 1, \ldots, F$ denote the coordinates of the automatically measured LA apex (denoted by vertex v_a in fig. 1) for the cardiac cycle having F frames. The measured $[x(f), y(f)]$ coordinates for frame f are assumed to follow a Gaussian noise model given by:

$$\left[\underbrace{x(f)\ y(f)}_{1\times 2} \right] = \sum_{j=0}^{2} B_j(f) \left[\underbrace{a_j\ b_j}_{1\times 2} \right] + \left[\underbrace{\eta_x(f)\ \eta_y(f)}_{1\times 2} \right], \quad (1)$$

where, $\{B_0(f) = 1, B_1(f) = f, B_2(f) = f^2\}$ is the basis set. $\eta_x(f) \sim \mathcal{N}(0, \sigma_x^2(f))$, $\eta_y(f) \sim \mathcal{N}(0, \sigma_y^2(f))$. $\sigma_x^2(f)$ is the variance of the noise for x-coordinate for frame f. $\sigma_y^2(f)$ is the variance of noise for y-coordinate for frame f. The Gaussian noise perturbing two different frames is assumed to be independent. We also assume that the x-coordinate and y-coordinate noise are independent. Let $\boldsymbol{\alpha}=(a_0, a_1, a_2)^T$ and $\boldsymbol{\beta}=(b_0, b_1, b_2)^T$ be the coefficients associated with x and y coordinates of the apex respectively. If all the frames of the cardiac cycle are taken into account and represented in a matrix form we have: $\mathbf{V}=[\mathbf{X}\ \mathbf{Y}]^{F\times 2}$, where $\mathbf{X} = [x(1), \ldots, x(F)]^T$ and $\mathbf{Y} = [y(1), \ldots, y(F)]^T$. Let F_{in} and F_{out} be the set of inlier and outlier frames, $\#F_{in} + \#F_{out} = F$, where F is the total number of frames in the cardiac cycle. $F_{com}=\{F_{in} \cup F_{out}\}$ is called the Combined Data. We also assume that $\#F_{out} \leq \frac{1}{4}\#F_{in}$. The above model can be represented in the matrix form as:

$$[\ \mathbf{V}\ ] = [\ \mathbf{XY}\ ]^{F\times 2} = [\mathbf{B}\psi]^{F\times 2} + [\ \boldsymbol{\eta}_x\ \boldsymbol{\eta}_y\ ]^{F\times 2} \quad (2)$$

where $\psi = [\ \boldsymbol{\alpha}\,\boldsymbol{\beta}\ ]^{3\times 2}$ are the coefficients associated with the x- and y-coordinates. $\mathbf{B}\psi$ is the matrix holding the true unperturbed coordinates of the cardiac cycle. $\mathbf{V}$ is the matrix holding the LA data since it consists of the observed apex or aortic coordinates for the complete cardiac cycle. $\mathbf{B}$ is the basis matrix for the given cardiac cycle: $\mathbf{B} = \begin{pmatrix} 1\ 1\ 1^2 \\ \cdot \\ 1\ f\ f^2 \\ \cdot \\ 1\ F\ F^2 \end{pmatrix}^{F\times 3}$. The distribution of the noise vector $\boldsymbol{\eta}_x$ $(F\times 1)$ for the cardiac cycle has mean 0 and covariance Σ_x, given as: $\boldsymbol{\eta}_x \sim \mathcal{N}(0, \Sigma_x)$. Similarly, the distribution of the noise vector $\boldsymbol{\eta}_y$ $(F\times 1)$ for the cardiac cycle has mean 0 and covariance Σ_y, given as: $\boldsymbol{\eta}_y \sim \mathcal{N}(0, \Sigma_y)$. The covariance

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matrix Σ_x for η_x is given as:

$$\Sigma_x^{F\times F} = \begin{pmatrix} \sigma_x^2(1) & 0 & . & . & . & . \\ 0 & . & 0 & . & . & . \\ . & 0 & . & . & . & . \\ . & . & . & \sigma_x^2(f) & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & 0 & \sigma_x^2(F) \end{pmatrix} \quad (3)$$

where the variance for frame f is given as:

$$\sigma_x^2(f) = \begin{cases} \sigma_{in}^2 \; for \; f \in F_{in} \\ \sigma_{out}^2 \; for \; f \in F_{out} \end{cases} \quad (4)$$

where, $\sigma_{out}^2 \gg \sigma_{in}^2$, i.e., the noise from to the outlier frames (F_{out}) is much larger than the noise from to the inlier frames (F_{in}). The covariance for y-coordinate (Σ_y) is defined in the same way as Σ_x. Note the Gaussian noise perturbing two different frames is assumed to be independent.

The problem is to estimate $[\hat{x}(f), \hat{y}(f)]$, $f = 1, \ldots, F$, the coordinates of LA apex or aortic points using an iterative least square robust procedure (IRLS). In matrix notation we estimate: $\hat{\mathbf{V}}=[\hat{\mathbf{X}}\ \hat{\mathbf{Y}}] = [\mathbf{B}\hat{\psi}]^{F\times 2}$ where, $\hat{\mathbf{X}} = [\hat{x}(1), \ldots, \hat{x}(F)]^T$ and $\hat{\mathbf{Y}} = [\hat{y}(1), \ldots, \hat{y}(F)]^T$. $\hat{\psi}(3\times 2)$ is the estimated coefficient matrix. The problem thus reduces to estimating $\hat{\psi}$ using an iterative least square robust procedure (IRLS).

III. Ruled Surface s_r and its Coefficients

Consider the ruled surface (see fig. 1) generated by the LA during the cardiac cycle. We model this ruled surface in the quadratic parametric form given in Eq. 1. Let the apex and aortic end vertices of the LA for a frame f in the cardiac cycle be $[x_a(f),\ y_a(f)]$ and $[x_b(f),\ y_b(f)]$ respectively. The ruled surface is mathematically given as:

$$s_r(f,\lambda) = (1-\lambda)\begin{bmatrix} x_a(f) \\ y_a(f) \\ f \end{bmatrix} + (\lambda)\begin{bmatrix} x_b(f) \\ y_b(f) \\ f \end{bmatrix} \quad (5)$$

where λ, $0 \le \lambda \le 1$, is the variable designating a position along the LA and $f = 1, \ldots, F$ is the frame number. Thus, the ruled surface equation is a function of the end coordinates of the LA which in turn is a function of $\psi = [\ \alpha\, \beta\ ]$, the apex coefficient matrix and $\zeta = [\ \gamma\, \delta\ ]$, the similarly defined aortic coefficient matrix. Since the estimation for the apex coefficients and aortic coefficients is similar, we use a generic (x, y) in the remainder of the paper, not distinguishing between (x_a, y_a) and (x_b, y_b).

IV. Estimation of robust coefficients

The coefficient matrix of the ruled surface is robustly estimated using the iterative re-weighted least square procedure in which low weights are assigned to the outliers which have high residual error and high weights are assigned to the inliers which have low residual error. This weighting has the effect of reducing the influence of the outliers on the estimated coefficients. At each iteration, a weighted least square problem is solved. The weight matrix for the first iteration is taken to be the identity matix. The weights for each successive iteration are a function of the residual errors of the previous iteration. The weights for the current iteration are a function of the normalized residuals of estimated x- and estimated y- apex coordinates from the previous iteration. Thus, given the matrices for the Combined Data $\mathbf{V}$, the basis matrix $\mathbf{B}$, the diagonal weight matrix $\mathbf{W}$, the weighted least squares problem is to determine $\hat{\psi}_w$ to minimize the weighted residual squared error function ϵ_w^2 defined by:

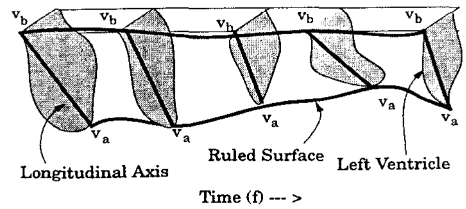

Fig. 1. Diagram showing the generation of the ruled surface from the motion of the LA in time. Longitudinal axes (or long axis) is the segment joining the starting vertex v_a and ending vertex v_b.

$$\epsilon_w^2 = \underbrace{\| \mathbf{W}\,(\mathbf{V} - \mathbf{B}\,\hat{\psi}_w)\ \|^2} \quad (6)$$

The estimated apex coordinate matrix $\hat{\mathbf{V}}$ is given by the product of $\mathbf{B}$ and the estimated coefficients $\hat{\psi}_w$ given as:

$$\hat{\mathbf{V}} = \mathbf{B}\ \hat{\psi}_w \quad (7)$$

To determine the $\hat{\psi}_w$ that minimizes ϵ_w^2, we take the partial derivative of ϵ_w^2 with respect to $\hat{\psi}_w$ and equate the result to 0. Since IRLS is an iterative process, at the c^{th} iteration we denote the weight matrix by $\mathbf{W}_c$. The robust coefficient vector $(\hat{\psi}_{robust})_c$ is given as:

$$\underbrace{(\hat{\psi}_{robust})_{c+1}}_{3\times 2} = \underbrace{(\mathbf{B}^T\,\mathbf{W}_c^T\,\mathbf{W}_c\,\mathbf{B}\,)^{-1}}_{3\times F} \times \underbrace{(\mathbf{B}^T\,\mathbf{W}_c^T\,\mathbf{W}_c\,\mathbf{V}\,)}_{F\times 2} \quad (8)$$

where, the weight matrix $\mathbf{W}$ given as:

$$\mathbf{W}^{F\times F} = Diag(w(1), w(2), w(3), ..., w(F)) \quad (9)$$

where, the weight $w(f)$ is given as:

$$w(f) = min\,(\frac{H_x + H_y}{|\,r_{nx}(f)| + |\,r_{ny}(f)\,|}\ ,\, 1) \quad (10)$$

where, H_x and H_y is a Huber's function for x- and y-coordinates and is given as a product of tuning constant (T_c) and the median of the normalized absolute residuals is given as:

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Table 1: Parameters used in longitudinal axis generation and fitting.

parameters	values
Small inlier noise (σ_{in}^2)	0.0, 1.0,2.0,3.0,4.0,5.0,6.0,...,10.0 (mm^2)
Large outlier noise (σ_{out}^2)	50.0, 70.0, 80.0,...,100.0 (mm^2)
Total number of outliers (n_o) per case	0%-20% (0 to 12 axes frames/any cycle)
Weights in IRLS fit	0 - 1
Tuning Constant (T_c)	$2.0 \leq T_c \leq 3.5$
Total number of trials (T_o)	100
Total number of studies (N)	40 (Clinical Data)

$$H_x = T_c \, med(| \, r_{nx}(f) \, |), 1 \leq f \leq F \qquad (11)$$
$$H_y = T_c \, med(| \, r_{ny}(f) \, |), 1 \leq f \leq F$$

Note "med" is the median computation over F frames of the cardiac cycle. The range of the tuning constant (T_c) is shown in the parameter table 1. The weights depend upon the absolute value of the normalized residual error. If the residual error is high, then low weights are assigned and vice versa. Eq. 8 can be solved using a singular value decomposition. Convergence is achieved in fewer than 10 iterations. Thus, the robust coefficients are: $\hat{\psi}_{robust}=(\hat{\psi}_{robust})_{10}$.

V. Experiment design

A. Data Generation Process

We have generated two kinds of data sets, Inlier Data sets, consisting of $(F-n_o)$ frames and Combined Data sets, consisting of F frames of which $(F-n_o)$ is the number of inlier frames and n_o is the number of outlier frames. n_o is an experimental parameter and is fixed between 0% to 20% of F. The set of n_o outlier frames $F_{out}=\{f_1, f_2, f_3, ..., f_{n_o}\}$ is selected at random by sampling the set $\{1, \ldots, F\}$ without replacement. Once we generate the sets of inlier and outlier frames, the synthetic Inlier Data and Combined Data is then generated using Eq. 1 and Eq. 2. The variation of Inlier (σ_{in}^2) and Outlier noise (σ_{out}^2) is shown in parameter table 1. An ordinary least squares fitting is applied to the Inlier Data. A robust iterative re-weighted least squares is applied to the Combined Data. The ratio of their errors measures the efficiency of the robust procedure.

B. Efficiency of a robust procedure (η^ψ)

The statistical efficiency of the robust procedure measures how well the robustly computed x- and y-coordinates of the end vertices for data *with outliers* compare with the non-robustly computed x- and y-coordinates of the apex vertices for data *without outliers*. This can be defined as the ratio of the *mean error* of Inlier Data having $(F - n_o)$ frames, to the *mean error* of the Combined Data having F frames. Thus the estimated statistical efficiency $\hat{\eta}^\psi$ is basically the ratio of $\hat{Q}_\psi^{in}$ to $\hat{Q}_\psi^{com}$: $\hat{\eta}^\psi = \frac{\hat{Q}_\psi^{in}}{\hat{Q}_\psi^{com}}$, where,

$$\hat{Q}_\psi^{in} = \hat{Q}_{\boldsymbol{\alpha}}^{in} + \hat{Q}_{\boldsymbol{\beta}}^{in} \;\; and \qquad (12)$$

$$\hat{Q}_{\boldsymbol{\alpha}}^{in} = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{1}{T_o}\sum_{t=1}^{T_o}\left[\frac{1}{F-n_o-3}\sum_{f\in F_{in}(t)}(x_n^{true}(f) - (\hat{x}_n^{in}(f))_t)^2\right]\right]$$

$$\hat{Q}_{\boldsymbol{\beta}}^{in} = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{1}{T_o}\sum_{t=1}^{T_o}\left[\frac{1}{F-n_o-3}\sum_{f\in F_{in}(t)}(y_n^{true}(f) - (\hat{y}_n^{in}(f))_t)^2\right]\right]$$

Similary, we can compute the denominator $\hat{Q}_{\boldsymbol{\alpha}}^{com}$, $\hat{Q}_{\boldsymbol{\beta}}^{com}$ and $\hat{Q}_\psi^{com}$ for the Combined Data.

C. Analytical error measure, AQ^{in} for Inlier Data

The analytical error measure of the ruled surface is computed by finding the expected value of the experimentally estimated error and is given as:

$$AQ_{\boldsymbol{\alpha}}^{in} = E\,[\hat{Q}_{\boldsymbol{\alpha}}^{in}] \qquad (13)$$

Substituting the value on the right, solving the equation, and then adding for the y coordinate we have:

$$AQ_\psi^{in} = \frac{1}{F-n_o-3}[\mathbf{X}_{in}^{true^T}(\mathbf{I}-\mathbf{P})^T(\mathbf{I}-\mathbf{P})\,\mathbf{X}_{in}^{true} + \mathbf{Y}_{in}^{true^T}(\mathbf{I}-\mathbf{P})^T(\mathbf{I}-\mathbf{P})\,\mathbf{Y}_{in}^{true} + 2\,\sigma_{in}^2\sum_{f\in F_{in}} p_{ff}] \qquad (14)$$

where, $\mathbf{W}$ equals identity, $\mathbf{I}$, and the projection operator is: $\mathbf{P} = \mathbf{B}\,\underbrace{(\mathbf{B}^T\mathbf{W}^T\mathbf{W}\,\mathbf{B})^{-1}\mathbf{B}^T\mathbf{W}^T\mathbf{W}}$.

VI. Experiments, Results & Discussions

A. Results & Discussions on Synthetic Data

The first four relationships are between the error measure of ruled surface and variance of the inlier noise and last relationship is between robust efficiency and number of outliers. We see from the curves (fig. 2), that as we increase the inlier perturbation, the error measure of the non-robust ruled surface also increases. This is because the inlier vertices are a functions of the input random perturbations. We also observe that with increasing inlier perturbation, the analytical error measure increases and has a difference of 0.09 mm compared to the experimental value when standard deviation of the inlier noise is 1 mm^2. Curve 3 shows that as we increase the inlier perturbation, the square root of joint error measure for Combined Inlier and Outlier Data using the non-robust ruled surface is approximately 10.5 mm. This is because the outlier perturbation is 50-100 times larger than the inlier perturbation and we are using the plain least squares estimate. As this Combined data undergoes the IRLS algorithm, the error measure drops drastically (0.5 mm to 2.2 mm) for inlier noise with standard deviations between 0.5 mm-1 mm. This demonstrates that the effect of outliers has been totally removed by assigning low weights to large outliers using IRLS fit procedure. In experiment 5 (see curve 3), as we increase the number of outliers (n_o) from 0, the efficiency ($\hat{\eta}^\psi$) drops slowly. In this experiment, the inlier noise (σ_{in}^2) is fixed to 1 mm^2, outlier noise (σ_{out}^2) is fixed to 100 mm^2, total trials (T_o) are fixed to 100, and tuning constant (T_c) is fixed to 3.0 for this parametric apex model. As we decrease or increase the tuning constant below or above 3.0 repectively, $\hat{\eta}^\psi$, drops sleeply at higher number of outliers. For the stability of the efficiency curve, T_c was selected as 3.0. We also observed that as we increase the outlier noise, keeping every thing else the same, a good tuning constant for our parameteric apex model lies in the range: 2.0 to 3.0.

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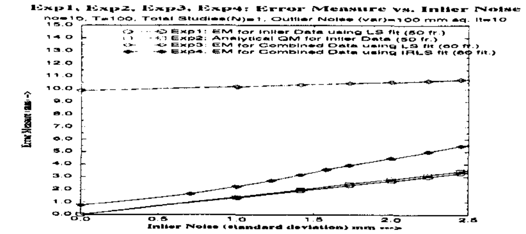

Fig. 2. Plot of joint mean error measure of the non-robust and robust ruled surface (in mm) with varying inlier noise (σ_{in}) (in mm), The parameters are: total trials (T_o), total studies (N) number of outliers (n_o) and the outlier noise (σ^2_{out}) is fixed to 100, 10, 10 and 100.0 mm^2 respectively. At σ_{in}=1, square root of $\hat{Q}^{com}_{\psi}$=1.25mm, square root of ($\hat{Q}^{in}_{\psi}$ - $\hat{Q}^{com}_{\psi}$)=0.09 mm.

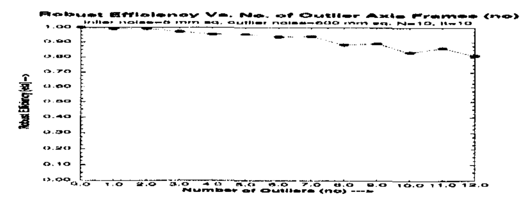

Fig. 3. Plot of efficiency ($\hat{\eta}^{\psi}$) vs. n_o, keeping T_o fixed to 100, σ^2_{in} fixed to 1 mm^2, σ^2_{out} fixed to 100 mm^2.

B. Results and Discussions on Clinical Data

We evaluate the robustly estimated apex location by comparing against the ground truth apex location as delineated by the cardiologist. In cardiological imaging it is a standard practice to divide the ventricle border into 100 equal length segments and compare the estimated position of the apex on the LVC as the vertex number, (referred also as serial number) which begins from the anterior aspect of aortic valve (i.e. in clock wise direction). We perform two types of experiments first, we compute the absolute position error of the estimated serial number to the true serial number as given by the cardiologist. In second experiment we compute the mean apex position error over all the patient studies N. If $(\hat{x}_n(f), \hat{y}_n(f))$ is the robust apex coordinates produced by the IRLS algorithm, for frame number f and patient study n, and if $(x_n^{true}(f), y_n^{true}(f))$ is the apex coordinates as given by the cardiologist for frame f study n, then the estimated distance between these coordinates $\hat{d}_n(f)$ is given as: $\hat{d}_n(f)$= $\sqrt{[\hat{x}_n(f) - x_n^{true}(f)]^2 + [\hat{y}_n(f) - y_n^{true}(f)]^2}$. Since we are interested in the serial number of the robust apex starting from the anterior aspect of aortic valve, this can be computed as the serial number *on true LVC* whose distance from the robust coordinates is minimum. The expression for the robust serial number ($\hat{s}_n(f)$) on true LVC is given as: $\hat{s}_n(f)$= $\{i \mid \min \hat{d}_n(f), 1 \leq i \leq P\}$, where, P is the total number of vertices on the LVC. We estimate the mean absolute position error over all the frames F and studies N by:

$$\bar{e} = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{1}{F}\sum_{f=1}^{F} \mid s_n^{true}(f) - \hat{s}_n(f) \mid\right] \qquad (15)$$

The probability of absolute position error is computed as follows. Let $n(e)$ be the number of times the absolute position error $e_n(f)$ occurs with an error e (say $e \leq 5$), then the probability ($\mathcal{P}(e)$) that the robust serial number and true serial number differ by e is given as: $\mathcal{P}(e) = \frac{\#n(e)}{NF}$. The results are plotted in fig. 4, left. As the error (e) inceases, the number of frames having error less than e drops. From the curve we see that about 60% frames have an error of 0 (same as the cardiologist location) and about 90% of the frames have an error of 1 which is considered as very close and excellent. The bias study plot is shown in fig. 4, right.

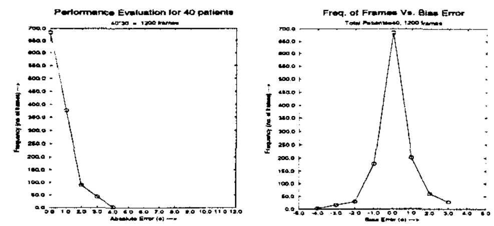

Fig. 4. Performance on 40 clinical studies consisting of 1200 frames. **Left**: Absolute position error, **Right**: Bias Study for apex position.

VII. Conclusion

We have developed a robust algorithm for fitting the the left ventricle longitudinal axis and estimating its apex coordinates. Our method shows that if the cardiac cycle has 15% outliers, the robust procedure has an efficiency of 90%. We validated our algorithms and showed a difference of 0.09 mm between the experimental and analytical errors when inlier noise is 1 mm^2. We also demostrated our algorithm on clinical data sets.

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