P. R. Krishnaiah and L. N. Kanal, eds., *Handbook of Statistics, Vol. 2*
North-Holland Publishing Company (1982) 399–415

18

Image Texture Survey*

Robert M. Haralick

1. Introduction

Texture is an important characteristic for the analysis of many types of images. It can be seen in all images from multi-spectral scanner images obtained from aircraft or satellite platforms (which the remote sensing community analyzes) to microscopic images of cell cultures or tissue samples (which the bio-medical community analyzes). Despite its important and ubiquity in image data, a formal approach or precise definition of texture does not exist. The texture discrimination techniques are, for the most part, ad-hoc. In this paper we survey, unify, and generalize some of the extraction techniques and models which investigators have been using to measure textural properties.

The image texture we consider is non-figurative and cellular. We think of this kind of texture as an organized area phenomenon. When it is decomposable, it has two basic dimensions on which it may be described. The first dimension is for describing the primitives out of which the image texture is composed, and the second dimension is for the description of the spatial dependence or interaction between the primitives of an image texture. The first dimension is concerned with tonal primitives or local properties, and the second dimension is concerned with the spatial organization of the tonal primitives.

Tonal primitives are regions with tonal properties. The tonal primitive can be described in terms such as the average tone, or maximum and minimum tone of its region. The region is a maximally connected set of pixels having a given tonal property. The tonal region can be evaluated in terms of its area and shape. The tonal primitive includes both its gray tone and tonal region properties.

An image texture is described by the number and types of its primitives and the spatial organization or layout of its primitives. The spatial organization may be random, may have a pairwise dependence of one primitive on a neighboring primitive, or may have a dependence of n primitives at a time. The dependence may be structural, probabilistic, or functional (like a linear dependence).

To characterize texture, we must characterize the tonal primitive properties as well as characterize the spatial inter-relationships between them. This implies that

*A full text of this paper by the same author appeared in *Proc. IEEE* **67** (5) (1979) 786–804, under the title "Statistical and Structural Approaches to Texture." Reprinted with permission.

texture-tone is really a two-layered structure, the first layer having to do with specifying the local properties which manifest themselves in tonal primitives and the second layer having to do with specifying the organization among the tonal primitives. We, therefore, would expect that methods designed to characterize texture would have parts devoted to analyzing each of these aspects of texture. In the review of the work done to date, we will discover that each of the existing methods tends to emphasize one or the other aspect and tends not to treat each aspect equally.

2. Review of the literature on texture models

There have been eight statistical approaches to the measurement and characterization of image texture: autocorrelation functions, optical transforms, digital transforms, textural edgeness, structural elements, spatial gray tone co-occurrence probabilities, gray tone run lengths, and auto-regressive models. An early review of some of these approaches is given by Hawkins (1970). The first three of these approaches are related in that they all measure spatial frequency directly or indirectly. Spatial frequency is related to texture because fine textures are rich in high spatial frequencies while coarse textures are rich in low spatial frequencies.

An alternative to viewing texture as spatial frequency distribution is to view texture as amount of edge per unit area. Coarse textures have a small number of edges per unit area. Fine textures have a high number of edges per unit area.

The structural element approach of Serra (1974) and Matheron (1967) uses a matching procedure to detect the spatial regularity of shapes called structural elements in a binary image. When the structural elements themselves are single resolution cells, the information provided by this approach is the autocorrelation function of the binary image. By using larger and more complex shapes, a more generalized autocorrelation can be computed.

The gray tone spatial dependence approach characterizes texture by the co-occurrence of its gray tones. Coarse textures are those for which the distribution changes only slightly with distance and fine textures are those for which the distribution changes rapidly with distance.

The gray level run length approach characterizes coarse textures as having many pixels in a constant gray tone run and fine textures as having few pixels in a constant gray tone run.

The auto-regressive model is a way to use linear estimates of a pixel's gray tone given the gray tones in a neighborhood containing it in order to characterize texture. For coarse textures, the coefficients will all be similar. For fine textures, the coefficients will have wide variation.

The power of the spatial frequency approach to texture is the familiarity we have with these concepts. However, one of the inherent problems is in regard to gray tone calibration of the image. The procedures are not invariant under even a linear translation of gray tone. To compensate for this, probability quantizing can be employed. But the price paid for the invariance of the quantized images under monotonic gray tone transformations is the resulting loss of gray tone precision in the quantized image. Weszka, Dyer and Rosenfeld (1976) compare the effectiveness of some of these techniques for terrain classification. They conclude that spatial frequency approaches perform significantly poorer than the other approaches.

The power of the structural element approach is that it emphasizes the shape aspects of the tonal primitives. Its weakness is that it can only do so for binary images.

The power of the co-occurrence approach is that it characterizes the spatial inter-relationships of the gray tones in a textural pattern and can do so in a way that is invariant under monotonic gray tone transformations. Its weakness is that it does not capture the shape aspects of the tonal primitives. Hence, it is not likely to work well for textures composed of large-area primitives.

The power of the auto-regressive linear estimator approach is that it is easy to use the estimator in a mode which synthesizes textures from any initially given linear estimator. In this sense, the auto-regressive approach is sufficient to capture everything about a texture. Its weakness is that the texture it can characterize are likely to consist mostly of micro-textures.

2.1. The autocorrelation function and texture

From one point of view, texture relates to the spatial size of the tonal primitives on an image. Tonal primitives of larger size are indicative of coarser textures; tonal primitives of smaller size are indicative of finer textures. The autocorrelation function is a feature which tells about the size of the tonal primitives.

We describe the autocorrelation function with the help of a thought experiment. Consider two image transparencies which are exact copies of one another. Overlay one transparency on top of the other and with a uniform source of light, measure the average light transmitted through the double transparency. Now translate one transparency relative to the other and measure only the average light transmitted through the portion of the image where one transparency overlaps the other. A graph of these measurements as a function of the (x, y) translated positions and normalized with respect to the $(0,0)$ translation depicts the two-dimensional autocorrelation function of the image transparency.

Let $I(u, v)$ denote the transmission of an image transparency at position (u, v). We assume that outside some bounded rectangular region $0 \leqslant u \leqslant L_x$ and $0 \leqslant v \leqslant L_y$ the image transmission is zero. Let (x, y) denote the x-translation and y-translation, respectively. The autocorrelation function for the image transparency d is formally defined by

$$\rho(x, y) = \frac{\dfrac{1}{(L_x - |x|)(L_y - |y|)} \displaystyle\int_{-\infty}^{\infty} \int I(u, v) I(u + x, v + y) \, du \, dv}{\dfrac{1}{L_x L_y} \displaystyle\int_{-\infty}^{\infty} \int I^2(u, v) \, du \, dv},$$

$$|x| < L_x \quad \text{and} \quad |y| < L_y.$$

If the tonal primitives on the image are relatively large, then the autocorrelation will drop off slowly with distance. If the tonal primitives are small, then the autocorrelation will drop off quickly with distance. To the extent that the tonal primitives are spatially periodic, the autocorrelation function will drop off and rise again in a periodic manner. The relationship between the autocorrelation function and the power spectral density function is well known: they are Fourier transforms of one another (Yaglom, 1962).

The tonal primitive in the autocorrelation model is the gray tone. The spatial organization is characterized by the correlation coefficient which is a measure of the linear dependence one pixel has on another.

An experiment was carried out by Kaizer (1955) to see of the autocorrelation function had any relationship to the texture which photointerpreters see in images. He used a series of seven aerial photographs of an Arctic region and determined the autocorrelation function of the images with a spatial correlator which worked in a manner similar to the one envisioned in our thought experiment. Kaizer assumed the autocorrelation function was circularly symmetric and computed in only as a function of radial distance. Then for each image, he found the distance d such that the autocorrelation function ρ at d took the value $1/e$; $\rho(d) = 1/e$.

Kaizer then asked 20 subjects to rank the seven images on a scale from fine detail to coarse detail. He correlated the rankings with the distances corresponding to the $(1/e)$th value of the autocorrelation function. He found a correlation coefficient of 0.99. This established that at least for his data set, the autocorrelation function and the subjects were measuring the same kind of textural features.

Kaizer noticed, however, that even though there was a high degree of correlation between $\rho^{-1}(1/e)$ and subject rankings, some subjects put first what $\rho^{-1}(1/e)$ put fifth. Upon further investigation, he discovered that a relatively flat background (indicative of high frequency or fine texture) can be interpreted as a fine textured or coarse textured area. This phenomena is not unusual and actually points out a fundamental characteristic of texture: it cannot be analyzed without a reference frame of tonal primitive being stated or implied. For any smooth gray tone surface there exists a scale such that when the surface is examined, it has no texture. Then as resolution increases, it takes on a fine texture and then a coarse texture. In Kaizer's situation, the resolution of his spatial correlator was not good enough to pick up the fine texture which some of his subjects did in an area which had a weak but fine texture.

2.2. Orthogonal transformations

Spatial frequency characteristics of two-dimensional images can be expressed by the autocorrelation function or by the power spectra of those images. Both may be calculated digitally and/or implemented in a real-time optical system.

Lendaris and Stanley (1969, 1970) used optical techniques to perform texture analysis on a data base of low altitude photographs. They illuminated small circular sections of those images and used the Fraunhoffer diffraction pattern to generate features for identifying photographic regions. The major discriminations of concern to these investigators were those of man-made versus natural scenes. The man-made category was further subdivided into roads, intersections of roads, buildings and orchards.

Feature vectors extracted from these diffraction patterns consisted of forty components. Twenty of the components were mean energy levels in concentric annular rings of the diffraction pattern and the other twenty components were mean energy levels in 9°-wedges of the diffraction pattern. Greater than 90% classification accuracy was reported using this technique.

Cutrona, Leith, Palermo and Porcello (1969) present a review of optical processing methods for computing the Fourier transform. Goodman (1968), Preston (1972) and Shulman (1970) also present in their books comprehensive reviews of Fourier optics. Swanlund (1971) discusses the hardware specifications for a system using optical techniques to perform texture analysis.

Gramenopolous (1973) used a digital Fourier transform technique to analyze aerial images. He examined subimages of 32×32 pixels and determined that for an (ERTS) image over Phoenix, spatial frequencies between 3.5 and 5.9 cycles/km contained most of the information required to discriminate among terrain types. An overall classification accuracy of 87% was achieved using image categories of clouds, water desert, farms, mountain, urban, river bed and cloud shadows. Horning and Smith (1973) used a similar approach to interpret aerial multispectral scanner imagery.

Bajscy (1972, 1973) and Bajscy and Lieberman (1974, 1976) computed the two-dimensional power spectra of a matrix of square image windows. They expressed the power spectrum in a polar coordinate system of radius (r) versus angle (a). As expected, they determined that directional textures tend to have peaks in the power spectrum along a line orthogonal to the principle direction of the texture. Blob-like textures tend to have peaks in the power spectrum at radii (r) comparable to the sizes of the blobs. This work also shows that texture gradients can be measured by determining the trends of relative maxima of radii (r) and angles (a) as a function of the position of the image window whose power spectrum is being analyzed. For example, as the power peaks along the radial direction tend to shift towards larger values of r, the image surface becomes more finely textured. In general, features based on Fourier power spectra have been shown to perform more poorly than features based on second order gray level co-occurrence statistics (Haralick, Shanmugam, and Dinstein, 1973) or those based on first order statistics of gray level differences (Weszka, Dyer and Rosenfeld, 1976). Presence of aperture effects has been hypothesized to account for part of the unfavorable performance by Fourier features compared to space-domain gray level statistics (Dyer and Rosenfeld, 1976), although experimental results indicate that this effect, if present, is minimal.

Transforms other than the Fourier Transform can be used for texture analysis. Kirvida (1976) compared the fast Fourier, Hadamard and Slant transforms for textural features on aerial images of Minnesota. Five classes (i.e., hardwood trees, conifers, open space, city and water) were studied using 8×8 subimages. A 74%

correct classification rate was obtained using only spectral information. This rate increased to 98.5% when textural information was also included in the analysis. These researchers reported no significant difference in the classification accuracy as a function of which transform was employed.

Pratt (1978) and Pratt, Faugeras and Gagalowitz (1978) suggest measuring texture by the coefficients of the linear filter required to decorrelate an image and by the first four moments of the gray level distribution of the decorrelated image. They have shown promising preliminary results.

The linear dependence which one image pixel has on another is well known and can be measured by the autocorrelation function. This linear dependence is exploited by the autoregression texture characterization and synthesis model developed by McCormick and Jayaramamurthy (1974) to synthesize textures. McCormick and Jayaramamurthy used the Box and Jenkins (1970) time series seasonal analysis method to estimate the parameters of a given texture. These estimated parameters and a given set of starting values were then used to illustrate that the synthesized texture was close in appearance to the given texture. Deguchi and Morishita (1978), Tou, Kao and Chang (1976) and Tou and Chang (1976) used similar techniques.

The autoregressive model for texture synthesis begins with a randomly generated noise image. Then, given any sequence of K synthesized gray level values in its immediately past neighborhood, the next gray level value can be synthesized as a linear combination of those values plus a linear combination of the previous L random noise values. The coefficients of these linear combinations are the parameters of the model. Texture analysis work based on this model requires the identification of these coefficient values from a given texture image.

2.3. Gray tone co-occurrence

Textural features can also be calculated from a gray level spatial co-occurrence matrix. The co-occurrence $P(i, j)$ of gray tone i and j for an image I is defined as the number of pairs of neighboring resolution cells (pixels) having gray levels i and j, respectively. The co-occurrence matrix can be normalized by dividing each entry by the sum of all of the entries in the matrix. Conditional probability matrices can also be used for textural feature extraction with the advantage that these matrices are not affected by changes in the gray level histogram of an image, only by changes in the topological relationships of gray levels within the image.

Apparently Julesz (1962) was the first to use co-occurrence statistics in visual human texture discrimination experiments. Darling and Joseph (1968) used statistics obtained from nearest-neighbor gray-level transition probability matrices to measure texture using spatial intensity dependence in satellite images taken of clouds. Bartels and Wied (1975), Bartels, Bahr and Wied (1969) and Wied, Bahr and Bartels (1970) used one-dimensional co-occurrence statistics for the analysis of cervical cells. Rosenfeld and Troy (1970), Haralick (1971) and Haralick, Shanmugan and Dinstein (1973) suggested the use of spatial co-occurrence for arbitrary distances and directions. Galloway (1975) used gray level run length statistics to measure texture. These statistics are computable from co-occurrence assuming that the image is generated by a Markov process. Chen and Pavlidis (1978) used the co-occurrence matrix in conjunction with a split and merge algorithm to segment an image at textural boundaries. Tou and Chang (1977) used statistics from the co-occurrence matrix, followed by a principal components eigenvector dimensionality reduction scheme (Karhunen–Loève expansion) to reduce the dimensionality of the classification problems.

Statistics which Haralick, Shanmugan and Dinstein (1973) computed from such co-occurrence matrices have been used to analyze textures in satellite images (Haralick and Shanmugan, 1974). An 89% classification accuracy was obtained. Additional applications of this technique include the analysis of microscopic images (Haralick and Shanmugan, 1973), pulmonary radiographs (Chien and Fu, 1974) and cervical cell, leukocyte and lymph node tissue section images (Pressman, 1976a, 1976b).

Haralick (1975) illustrates a way to use co-occurrence matrices to generate an image in which the value at each resolution cell is a measure of the texture in the resolution cell's neighborhood. All of these studies produced reasonable results on different textures. Conners and Harlow (1976) concluded that this spatial gray level dependence technique is more powerful than spatial frequency (power spectra), gray level difference (gradient) and gray level run length methods (Galloway, 1975) of texture quantitation.

2.4. *Mathematical morphology*

A structural element and filtering approach to texture analysis of binary images was proposed by Matheron (1967) and Serra and Verchery (1973). This approach requires the definition of a structural element (i.e., a set of pixels constituting a specific shape such as a line or square) and the generation of binary images which result from the translation of the structural element through the image and the erosion of the image by the structural element. The textural features can be obtained from the new binary images by counting the number of pixels having the value 1. This mathematical morphology approach of Serra and Matheron is the basis of the Leitz Texture Analyser (LTA) (Muller and Hunn, 1974; Muller, 1974; Serra, 1974). A broad spectrum of applications has been found for this quantitative analysis of microstructures method in materials science and biology.

Watson (1975) summarizes this approach to texture analysis and we now give a precise description. Let H, a subset of resolution cells, be the structural element. We define the translate of H by row column coordinates (r,c) as $H(r,c)$ where

$$H(r,c)=\{(i,j)\mid \text{for some } (r',c')\in H, x=r+r', c=c+c'\}.$$

Then the erosion of F by the structural element H, written $F\ominus H$, is defined as

$$F\ominus H=\{(m,n)\mid H(m,n)\subseteq F\}.$$

The eroded image J obtained by eroding F with structural element H is a binary image where pixels take the value 1 for all resolution cells in $F\ominus H$. Textural properties can be obtained from the erosion process by appropriately parameterizing the structural element (H) and determining the number of elements of the erosion as a function of the parameter's value. Theoretical properties of the erosion operator as well as other operators are presented by Matheron (1975), Serra (1978) and Lantuejoul (1978). The importance of this approach to texture analysis is that properties obtained by the application of operators in mathematical morphology can be related to physical properties of the materials imaged.

2.5. *Gradient analysis*

Rosenfeld and Troy (1970) and Rosenfeld and Thurston (1971) regard texture in terms of the amount of 'edge' per unit image area. An edge can be detected by a variety of local mathematical operators which essentially measure some property related to the gradient of the image intensity function. Rosenfeld and Thurston used the Roberts gradient and then computed, as a measure of texture for any image window, the average value of the Roberts gradient taken over all of the pixels in the window. Sutton and Hall (1972) extend this concept by measuring the gradient as a function of the distance between pixels. An 80% classification accuracy was achieved by applying this textural measure in a pulmonary disease identification experiment.

Related approaches include Triendl (1972) who, smoothes the image using 3×3 neighborhoods, then applies a 3×3 digital Laplacian operator and finally smoothes the image with an 11×11 window. The resulting texture parameters obtained from the frequency filtered image can be used as a discriminatory textural feature. Hsu (1977) determines edgeness by computing variance-like measures for the intensities in a neighborhood of pixels. He suggests the deviation of the intensities in a pixel's neighborhood from both the intensity of the central pixel and from the average intensity of the neighborhood. The histogram of a gradient image was used to generate textural parameters by Landeweerd and Gelsema (1978) to measure texture properties in the nuclei of leukocytes. Rosenfeld (1975) generates an image whose intensity is proportional to the edge per unit area of the original image. This transformed image is then further processed by gradient transformations prior to textural feature extraction.

For example, mosaic texture models tessellate a picture into regions and assign a gray level to the region according to a specified probability density function (Schacter, Rosenfeld and Davis, 1978). Among the kinds of mosaic models are the Occupancy Model (Miles, 1970), Johnson–Mehl Model (Gilbert, 1962), Poisson Line Model (Miles, 1969) and Bombing Model (Switzer, 1967). The mosaic texture models seem readily adaptable to numerical analysis and their properties seem amenable to mathematical analysis.

3. Structural approaches to texture models

Pure structural models of texture presume that textures consist of primitives which appear in quasi-periodic spatial arrangements. Descriptions of these primitives and their placement rules can be used to describe textures (Rosenfeld and Lipkin, 1970). The identification and location of a particular primitive in an image may be probabilistically related to the identification and distribution of primitives in its neighborhood.

Carlucci (1972) suggests a texture model using primitives of line segments, open polygons and closed polygons in which the placement rules are given syntactically in a graph-like language. Zucker (1976a, 1976b) conceives of a real texture to be the distortion of an ideal texture. Zucker's model, however, is more of a competance based model than a performance model. Lu and Fu (1978) and Tsai and Fu (1978) use a syntactic approach to texture.

In the remainder of this section, we discuss some structural-statistical approaches to texture models. The approach is structural in the sense that primitives are explicitly defined. The approach is statistical in that the spatial interaction, or lack of it, between primitives is measured by probabilities.

We classify textures as being weak textures, or strong textures. Weak textures are those which have weak spatial-interaction between primitives. To distinguish between them it may be sufficient to only determine the frequency with which the variety of primitive kinds occur in some local neighborhood. Hence, weak texture measures account for many of the statistical textural features. Strong textures are those which have non-random spatial interactions. To distinguish between them it may be sufficient to only determine, for each pair of primitives, the frequency with which the primitives co-occur in a specified spatial relationship. Thus, our discussion will center on the variety of ways in which primitives can be defined and the ways in which spatial relationships between primitives can be defined.

3.1. Primitives

A primitive is a connected set of resolution cells characterized by a list of attributes. The simplest primitive is the pixel with its gray tone attribute. Sometimes it is useful to work with primitives which are maximally connected sets of resolution cells having a particular property. An example of such a primitive is a maximally connected set of pixels all having the same gray tone or all having the same edge direction.

Gray tones and local properties are not the only attributes which primitives may have. Other attributes include measures of shape of connected region and homogeneity of its local property. For example, a connected set of resolution cells can be associated with its length or elongation of its shape or the variance of its local property.

3.2. Spatial relationships

Once the primitives have been constructed, we have available a list of primitives, their center coordinates, and their attributes. We might also have available some topological information about the primitives, such as which are adjacent to which. From this data, we can select a simple spatial relationship such as *adjacency* of primitives or *nearness* of primitives and count how many primitives of each kind occur in the specified spatial relationship.

More complex spatial relationships include closest distance or closest distance within an angular window. In this case, for each kind of primitive situated in the texture, we could lay expanding circles around it and locate the shortest distance between it and every other kind of primitive. In this case our co-occurrence frequency is three-dimensional, two dimensions for primitive kind and one dimension for shortest distance. This can be dimensionally reduced to two dimensions by considering only the shortest distance between each pair of like primitives.

3.3. Weak texture measures

Tsuji and Tomita (1973) and Tomita, Yachida, and Tsuji (1973) describe a structural approach to weak texture measures. First a scene is segmented into atomic regions based on some tonal property such as constant gray tone. These regions are the primitives. Associated with each primitive is a list of properties such as size and shape. Then they make a histogram of size property or shape property over all primitives in the scene. If the scene can be decomposed into two or more regions of homogeneous texture, the histogram will be multi-modal. If this is the case, each primitive in the scene can be tagged with the mode in the histogram to which it belongs. A region growing/cleaning process on the tagged primitives yields the homogeneous textural region segmentation.

If the initial histogram modes overlap too much, a complete segmentation may not result. In this case, the entire process can be repeated with each of the then so far found homogeneous texture region segments. If each of the homogeneous texture regions consists of mixtures of more than one type of primitive, then the procedure may not work at all. In this case, the technique of co-occurrence of primitive properties would have to be used.

Zucker, Rosenfeld and Davis (1975) used a form of this technique by filtering a scene with a spot detector. Non-maxima pixels on the filtered scene were thrown out. If a scene has many different homogeneous texture regions, the histogram of the relative max spot detector filtered scene will be multi-modal. Tagging the maxima with the modes they belong to and region growing/cleaning thus produced the segmented scene.

The idea of the constant gray level regions of Tsuji and Tomita or the spots of Zucker, Rosenfeld, and Davis can be generalized to regions which are peaks, pits, ridges, ravines, hillsides, passes, breaks, flats and slopes (Toriwaki and Fukumura, 1978; Penucker and Douglas, 1975). In fact, the possibilities are numerous enough that investigators doing experiments will have a long working period before understanding will exhaust the possibilities. The next three subsections review in greater detail some specific approaches and suggest some generalizations.

3.3.1. Edge per unit area

Rosenfeld and Troy (1970) and Rosenfeld and Thurston (1971) suggested the amount of edge per unit area for a texture measure. The primitive here is the pixel and its property is the magnitude of its gradient. The gradient can be calculated by any one of the gradient neighborhood operators. For some specified window centered on a given pixel, the distribution of gradient magnitudes can then be determined. The mean of this distribution is the amount of edge per unit area associated with the given pixel. The image in which each pixel's value is edge per unit area is actually a defocussed gradient image. Triendl (1972) used a defocussed Laplacian image. Sutton and Hall (1972) used such a measure for the automatic classification of pulmonary disease in chest X-rays.

Ohlander (1975) used such a measure to aid him in segmenting textured scenes. Rosenfeld (1975) gives an example where the computation of gradient direction on a defocussed gradient image is an appropriate feature for the direction of texture gradient. Hsu (1977) used a variety of gradient-like measures.

3.3.2. Run lengths

The gray level run lengths primitive in its one-dimensional form is maximal collinear connected set of pixels all having the same gray level. Properties of the primitive can be length of run, gray level, and angular orientation of the run. Statistics of these properties were used by Galloway (1975) to distinguish between textures.

In the two-dimensional form, the gray level run length primitive is a maximal-connected set of pixels all having the same gray level. These maximal homogeneous sets have properties such as number of pixels maximum or minimum diameter, gray level, angular orientation of maximum or minimum diameter. Maleson et al. (1977) have done some work related to maximal homogeneous sets and weak textures.

3.3.3. Relative extrema density

Rosenfeld and Troy (1970) suggest the number of extrema per unit area for a texture measure. They define extrema in a purely local manner allowing plateaus to be considered extrema. Ledley (1972) also suggests computing the number of extrema per unit area as a texture measure.

Mitchell, Myers and Boyne (1977) suggest the extrema idea of Rosenfeld and Troy except they proposed to use true extrema and to operate on a smoothed image to eliminate extrema due to noise. See also the work by Carlton and Mitchell (1977) and Ehrich and Foith (1976, 1978).

One problem with simply counting all extrema in the same extrema plateau as extrema is that extrema per unit area is not sensitive to the difference between a region having few large plateaus of extrema and many single pixel extrema. The solution to this problem is to only count an extrema plateau once. This can be achieved by locating some central pixel in the extrema plateau and marking it as the extrema associated with the plateau. Another way of achieving this is to associate a value $1/N$ for every extremum in a N-pixel extrema plateau.

In the one-dimensional case there are two properties that can be associated with every extremum: its height and its width. The height of a maximum can be defined as the difference between the value of the maximum and the highest adjacent minimum. The height (depth) of a minimum can be defined as the difference between the value of the minimum and the lowest adjacent maximum. The width of a maximum is the distance between its two adjacent minima. The width of a minimum is the distance between its two adjacent maxima.

Two-dimensional extrema are more complicated than one-dimensional extrema. One way of finding extrema in the full two-dimensional sense is by the iterated use of some recursive neighborhood operators propagating extrema values in an appropriate way. Maximally connected areas of relative extrema may be areas of single pixels or may be plateaus of many pixels. We can mark each pixel in a relative extrema region of size N with the value h indicating that it is part of a relative extremum having height h or mark it with the value h/N indicating its contribution to the relative extrema area. Alternatively, we can mark the most centrally located pixel in the relative extrema region with the value h. Pixels not marked can be given the value 0. Then for any specified window centered on a given pixel, we can add up the values of all pixels in the window. This sum divided by the window size is the average height of extrema in the area. Alternatively we could set h to 1 and the sum would be the number of relative extrema per unit area to be associated with the given pixel.

Going beyond the simple counting of relative extrema, we can associate properties to each relative extremum. For example, given a relative maximum, we can determine the set of all pixels reachable only by the given relative maximum and not by any other relative maximum by monotonically decreasing paths. This set of reachable pixels is a connected region and forms a mountain. Its border pixels may be relative minima or saddle pixels.

The relative height of the mountain is the difference between its relative maximum and the highest of its exterior border pixels. Its size is the number of pixels which constitute it. Its shape can be characterized by features such as elongation, circularity, and symmetric axis. Elongation can be defined as the ratio of the larger to small eigenvalue of the 2×2 second moment matrix obtained from the $\binom{x}{y}$ coordinates of the border pixels (Bachi, 1973; Frolov, 1975). Circularity can be defined as the ratio of the standard deviation to the mean of the radii from the region's center to its border (Haralick, 1975). The symmetric axis feature can be determined by thinning the region down to its skeleton and counting the number of pixels in the skeleton. For regions which are elongated it may be important to measure the direction of the elongation or the direction of the symmetric axis.

3.4. Strong texture measures and generalized co-occurrence

Strong texture measures take into account the co-occurrence between texture primitives. On the basis of Julesz (1975) it is probably the case that the most important interaction between texture primitives occurs as a two-way interaction. Textures with identical second and lower order interactions but with different higher order interactions tend to be visually similar.

The simplest texture primitive is the pixel with its gray tone property. Gray tone co-occurrence between neighboring pixels was suggested as a measure of texture by a number of researchers as discussed in Section 2.6. All the studies mentioned there achieved a reasonable classification accuracy of different textures using co-occurrences of the gray tone primitive.

The next more complicated primitive is a connected set of pixels homogeneous in tone (Tsuji and Tomita, 1973). Such a primitive can be characterized by size, elongation, orientation, and average gray tone. Useful texture measures include co-occurrence of primitives based on relationships of distance or adjacency. Maleson et al. (1977) suggest using region growing techniques and ellipsoidal approximations to define the homogeneous regions and degree of co-linearity as one basis of co-occurrence. For example, for all primitives of elongation greater than a specified threshold we can use the angular orientation of each primitive with respect to its closest neighboring primitive as a strong measure of texture.

Relative extrema primitives were proposed by Rosenfeld and Troy (1970), Mitchell, Myers and Boyne (1977), Ehrich and Foith (1976), Mitchell and Carlton (1977), and Ehrich and Foith (1978). Co-occurrence between relative extrema was suggested by Davis et al. (1978). Because of their invariance under any monotonic gray scale transformation, relative extrema primitives are likely to be very important.

It is possible to segment an image on the basis of relative extrema (for example, relative maxima) in the following way: label all pixels in each maximally connected relative maxima plateau with a unique label. Then label each pixel with the label of the relative maximum that can reach it by a monotonically decreasing path. If more than one relative maximum can reach it by a monotonically decreasing path, then label the pixel with a special label 'c' for common. We call the regions so formed the descending components of the image.

Co-occurrence between properties of the descending components can be based on the spatial relationship of adjacency. For example, if the property is size, the co-occurrence matrix could tell us how often a descending component of size s_1 occurs adjacent to or nearby to a descending component of size s_2 or of label 'c'.

To define the concept of generalized co-occurrence, it is necessary to first decompose an image into its primitives. Let Q be the set of all primitives on the image. Then we need to measure primitive properties such as mean gray tone, variance of gray tones, region, size, shape, etc. Let T be the set of primitive properties and f be a function assigning to each primitive in Q a property of T. Finally, we need to specify a spatial relation between primitives such as distance or adjacency. Let $S \subseteq Q \times Q$ be the binary relation pairing all primitives which satisfy the spatial relation. The generalized co-occurrence matrix P is defined by

$$P(t_1, t_2) = \frac{\#\{(q_1, q_2) \in S \mid f(q_1) = t_1 \text{ and } f(q_2) = t_2\}}{\#S}.$$

$P(t_1, t_2)$ is just the relative frequency with which two primitives occur with specified spatial relationship in the image, one primitive having property t_1 and the other primitive having property t_2.

Zucker (1974) suggests that some textures may be characterized by the frequency distribution of the number of primitives any primitive has related to it. This probability $p(k)$ is defined by

$$p(k) = \frac{\#\{(q \in Q \mid \#S(q) = k\}}{\#Q}.$$

Although this distribution is simpler than co-occurrence, no investigator appears to have used it in texture discrimination experiments.

4. Conclusion

We have surveyed the image processing literature on the various approaches and models investigators have used for textures. For microtextures, the statistical approach seems to work well. The statistical approaches have included autocorrelation functions, optical transforms, digital transforms, textural edgeness, structural elements, gray tone co-occurrence, and autoregressive models. Pure structural approaches based on more complex primitives than gray tone seems not to be widely used. For macro-textures, investigators seem to be moving in the direction of using histograms of primitive properties and co-occurrence of primitive properties in a structural-statistical generalization of the pure structural and statistical approaches.

References

Bachi, R. (1973). Geostatistical analysis of territories. *Proc. 39th Session – Bull. Internat. Statist. Institute*. Vienna.

Bajcsy, R. (1972). Computer identification of textured visual scenes. Stanford Univ., Palo Alto, CA.

Bajcsy, R. (1973). Computer description of textured surfaces. *Third Internat. Joint Conf. on Artificial Intelligence*, 572–578. Stanford, CA.

Bajcsy, R. and L. Lieberman (1974). Computer description of real outdoor scenes. *Proc. Second Internat. Joint Conference on Pattern Recognition*, 174–179. Copenhagen, Denmark.

Bajcsy, R. and L. Lieberman (1976). Texture gradient as a depth cue. *Comput. Graphics Image Process.* **5** (1) 52–67.

Bartels, P., G. Bahr and G. Weid (1969). Cell recognition from line scan transition probability profiles. *Acta Cytol.* **13**, 210–217.

Bartels, P. H. and G. L. Wied (1975). Extraction and evaluation of information from digitized cell images. In: *Mammalian Cells: Probes and Problems*, 15–28. U.S. NTIS Technical Information Center, Springfield, VA.

Box, J. E. and G. M. Jenkins (1970). *Time Series Analysis*. Holden-Day, San Francisco, CA.

Carlton, S. G. and O. Mitchell (1977). Image segmentation using texture and grey level. In: *Pattern Recognition and Image Processing Conference*, 387–391. Troy, NY.

Carlucci, L. (1972). A formal system for texture languages. *Pattern Recognition* **4**, 53–72.

Chen, P. and T. Pavlidis (1978). Segmentation by texture using a co-occurrence matrix and a split-and-merge algorithm. Tech. Rept. 237. Princeton Univ., Princeton, NJ.

Chien, Y. P. and K. S. Fu (1974). Recognition of x-ray picture patterns. *IEEE Trans. Systems Man Cybernet.* **4** (2) 145–156.

Conners, R. W. and Ch. A. Harlow (1976). Some theoretical considerations concerning texture analysis of radiographic images. *Proc. 1976 IEEE Conf. on Decision and Control*. Clearwater Beach, FL.

Cutrona, L. J., E. N. Leith, C. J. Palermo and L. J. Porcello (1969). Optical data processing and filtering systems. *IRE Trans. Inform. Theory* **15** (6) 386–400.

Darling, E. M. and R. D. Joseph (1968). *IEEE Trans. Systems Man Cybernet.* **4**, 38–47.

Davis, L., S. Johns and J. K. Aggarwal (1978). Texture analysis using generalized co-occurrence matrices. *Pattern Recognition and Image Processing Conf.* Chicago, IL.

Deguchi, K. and I. Morishita (1978). Texture characterization and texture-based image partitioning using two-dimensional linear estimation techniques. *IEEE Trans. Comput.* **27** (8) 739–745.

Dyer, C. and A. Rosenfeld (1976). Fourier texture features: suppression of aperature effects. *IEEE Trans. Systems Man Cybernet.* **6** (10) 703–706.

Ehrich, Roger and J. P. Foith (1976). Representation of random waveforms by relational trees. *IEEE Trans. Comput.* **26** (7) 726–736.

Ehrich, R. and J. P. Foith (1978). Topology and semantics of intensity arrays. In: Hanson and Roseman, eds., *Computer Vision*. Academic Press, New York.

Frolov, Y. S. (1975). Measuring the shape of geographical phenomena: a history of the issue. *Soviet Geography: Review and Translation* **16** (10) 676–687.

Galloway, M. M. (1975). Texture analysis using gray level run lengths. *Comput. Graphics and Image Process.* **4** 172–179.

Gilbert, E. (1962). Random subdivisions of space into crystals. *Ann. Math. Statist.* **33**, 958–972.

Goodman, J. W. (1968). *Introduction to Fourier Optics*. McGraw-Hill, New York.

Gramenopoulos, N. (1973). Terrain type recognition using ERTS-1 MSS images. *Symp. on Significant Results Obtained from the Earth Resources Technology Satellite*, NASA SP-327, pp. 1229–1241.

Haralick, R. M. (1971). A texture-context feature extraction algorithm for remotely sensed imagery. *Proc. 1971 IEEE Decision and Control Conf.*, 650–657. Gainesville, FL.

Haralick, R. M. (1975). A textural transform for images. *Proc. IEEE Conf. on Computer Graphics, Pattern Recognition and Data Structure*. Beverly Hills, CA.

Haralick, R. M. and K. Shanmugam (1973). Computer classification of reservoir sandstones. *IEEE Trans. Geosci. Electronics* **11** (4) 171–177.

Haralick, R. M. and K. Shanmugam (1974). Combined spectral and spatial processing of ERTS imagery data. *J. Remote Sensing of the Environment* **3**, 3–13.

Haralick, R. M. and K. Shanmugam (1973). Textural features for image classification. *IEEE Trans. Systems Man Cybernet.* **3** (6) 610–621.

Hawkins, J. K. (1970). Textural properties for pattern recognition In: B. S. Lipkin and A. Rosenfeld, eds., *Picture Processing and Psychopictorics*. Academic Press, New York.

Horning, R. J. and J. A. Smith (1973). Application of Fourier analysis to multi-spectral/spatial recognition. *Management and Utilization of Remote Sensing Data ASP Symp*. Sioux Falls, SD.

Hsu, S. (1977). A texture-tone analysis for automated landuse mapping with panchromatic images. *Proc. Amer. Soc. for Photogrammetry*, 203–215.

Julesz, B. (1962). Visual pattern discrimination. *IRE Trans. Inform. Theory* **8** (2) 84–92.

Julesz, B. (1975). Experiments in visual perception of texture. *Sci. Amer.* **232**, 34–43.

Kaizer, H. (1955). A quantification of textures on aerial photographs. Tech. Note 121, AD 69484. Boston Univ. Res. Labs.

Kirvida, L. (1976). Texture measurements for the automatic classification of imagery. *IEEE Trans. Electromagnetic Compatibility* **18** (1) 38–42.

Landerweerd, G. H. and E. S. Gelsema (1978). The use of nuclear texture parameters in the automatic analysis of leukocytes. *Pattern Recognition* **10**, 57–61.

Lantuejoul, C. (1978). Grain dependence test in a polycristalline ceramic. In: J. L. Chernant, ed., *Quantitative Analysis of Microstructures in Materials Science, Biology and Medicine*, 40–50. Riederer, Stuttgart.

Ledley, R. S. (1972). Texture problems in biomedical pattern recognition. *Proc. 1972 IEEE Conf. on Decision and Control and the 11th Symp. on Adaptive Processes.* New Orleans, LA.

Lendaris, G. and G. Stanley (1969). Diffraction pattern sampling for automatic pattern recognition. *SPIE Pattern Recognition Studies Seminar Proceedings*, 127–154.

Lendaris, G. G. and G. L. Stanley (1970). Diffraction pattern samplings for automatic pattern recognition. *Proc. IEEE* **58** (2) 198–216.

Lu, S. Y. and K. S. Fu (1978). A syntactic approach to texture analysis. *Comput. Graphics and Image Process.* **7**, 303–330.

Maleson, J., C. Brown and J. Feldman (1977). Understanding natural texture. University of Rochester, Rochester, NY.

Matheron, G. (1967). *Elements pour une Théorie des Milieux Poreux*. Masson, Paris.

Matheron, G. (1975). *Random Sets and Integral Geometry*. Wiley, New York.

McCormick, B. H. and S. N. Jayaramamurthy (1974). Time series model for testure sythesis. *Internat. J. Comput. Informat. Sci.* **3** (4) 329–343.

Miles, R. (1969). Random polygons determined by random lines in the plane. *Proc. Nat. Acad. Sci. U.S.A.* **52**, 901–907; 1157–1160.

Miles, R. (1970). On the homogeneous planar poisson point-process. *Math. Biosci.* **6**, 85–127.

Mitchell, O., Ch. Myers and W. Boyne (1977). A max-min measure for image texture analysis. *IEEE Trans. Comput.* **25** (4) 408–414.

Mitchell, O. R. and S. G. Carlton (1977). Image segmentation using a local extrema texture measure. *Pattern Recognition* **10**, 205–210.

Muller, W. (1974). The leitz-texture-analyzing system. *Lab. Appl. Microscopy, Sci. and Tech. Inform.*, Suppl. I. **4**, 101–136. Wetzler, West Germany.

Muller, W. and W. Hunn (1974). Texture Analyzer System. *Industrial Res.*, 49–54.

Ohlander, R. (1975). Analysis of natural scenes. Ph.D. dissertation. Carnegie-Mellon Univ., Pittsburgh, PA.

Peucker, T. and D. Douglas (1975). Detection of surface-specific points by local parallel processing of discrete terrain elevation data. *Comput. Graphics and Image Process.* **4** (4) 375–387.

Pratt, W. K. (1978). Image feature extraction. *Digital Image Processing*, 471–513.

Pratt, W. K., O. D. Faugeras and A. Gagalowicz (1978). Visual discrimination of stochastic texture fields. *IEEE Trans. Systems Man Cybernet.* **8** (11) 796–804.

Pressman, N. J. (1976). Markovian analysis of cervical cell images. *J. Histochem. Cytochem.* **24** (1) 138–144.

Pressman, N. J. (1976b). Optical texture analysis for automated cytology and histology: A Markovian approach. Lawrence Livermore Lab. Rept. UCRL-52155. Livermore, CA.

Preston, K. (1972). *Coherent Optical Computers*. McGraw-Hill, New York.

Rosenfeld, A. (1975). A note on automatic detection of texture gradients. *IEEE Trans. comput.* **23** (10) 988–991.

Rosenfeld, A. and B. S. Lipkin, eds. (1970). *Picture Processing and Psychopictorics*. Academic Press, New York.

Rosenfeld, A. and M. Thurston (1971). Edge and curve detection for visual scene analysis. *IEEE Trans. Comput.* **20** (5) 562–569.

Rosenfeld, A. and E. Troy (1970). Visual texture analysis. Univ. of Maryland, College Park, MD; Tech. Rept. 70-116; ibid., *Conf. Record Symp. on Feature Extraction and Selection in Pattern Recognition*, IEEE Publ. 70C-51C (1970) 115–124. Argonne, IL.

Schacter, B. J., A. Rosenfeld and L. S. Davis (1978). Random mosaic models for textures. *IEEE Trans. Systems Man Cybernet.* **8** (9) 694–702.

Serra, J. (1974). Theoretical bases of the leitz texture analysis system. *Leitz Sci. Tech. Inform.*, Suppl. **1** (4) 125–136. Wetzlar, Germany.

Serra, J. (1978). One, two, three,..., infinity. In: J. L. Chernant, ed., *Quantitative Analysis of Microstructures in Materials Science, Biology and Medicine*, 9–24. Riederer, Stuttgart.

Serra, J. and G. Verchery (1973). Mathematical morphology applied to fibre composite materials. *Film Sci. Technol.* **6**, 141–158.

Shulman, A. R. (1970) *Optical Data Processing*. Wiley, New York.

Sutton, R. and E. Hall (1972). Texture measures for automatic classification of pulmonary disease. *IEEE Trans. Comput.* **21** (1) 667–676.

Swanlund, G. D. (1971). Design requirements for texture measurements. *Proc. Two Dimensional Digital Signal Processing Conf.* Univ. of Missouri, Columbia, MO.

Switzer, P. (1967). Reconstructing patterns for sample data. *Ann. Math. Statist.* **38**, 138–154.
Tomita, F. M. Yachida and S. Tsuji (1973). Detection of homogeneous regions by structural analysis. *Proc. Third Internat. Joint Conf. on Artificial Intelligence*, 564–571. Stanford Univ., Stanford, CA.
Toriwaki, J. and T. Fukumura (1978). Extraction of structural information from grey pictures. *Comput. Graphics and Image Process.* **7** (1) 30–51.
Tou, J. T. and Y. S. Chang (1976). An approach to texture pattern analysis and recognition. *Proc. 1976 IEEE Conf. on Decision and Control.* Clearwater Beach, FL.
Tou, J. T. and Y. S. Chang (1977). Picture understanding by machine via textural feature extraction. *Proc. 1977 IEEE Conf. on Pattern Recognition and Image Processing.* Troy, NY.
Tou, J. T., D. B. Kao and Y. S. Chang (1976). Pictorial texture analysis and synthesis. *Third Internat. Joint Conf. on Pattern Recognition*. Coronado, CA.
Triendl, E. E. (1972). Automatic terrain mapping by texture recognition. *Proc. Eighth Internat. Symp. on Remote Sensing of Environment.* Environmental Research Institute of Michigan Ann Arbor, MI.
Tsai, W. H. and K. S. Fu (1978). Image segmentation and recognition by texture discrimination: A syntactic approach. *Fourth Internat. Joint Conf. on Pattern Recognition.* Tokyo, Japan.
Tsuji, S. and F. Tomita (1973). A structural analyzer for a class of textures. *Comput. Graphics and Image Process.* **2**, 216–231.
Watson, G. S. (1975). *Geological Society of America Memoir* **142**, 367–391.
Weszka, J., C. Dyer and A. Rosenfeld (1976). A comparitive study of texture measures for terrain classification. *IEEE Trans. Systems Man Cybernet.* **6** (4) 269–285.
Wied, G., G. Bahr and P. Bartels (1970). Automatic analysis of cell images. In: Wied and Bahr, eds., *Automated Cell Identification and Cell Sorting.* Academic Press, New York.
Yaglom, A. M. (1962). *Theory of Stationary Random Functions.* Prentice-Hall, Englewood Cliffs, NJ.
Zucker, S. W. (1976a). Toward a model of texture. *Comput. Graphics and Image Process.* **5**, 190–202.
Zucker, S. W. (1976b). On the structure of texture. *Perception* **5**, 419–436.
Zucker, S. (1974). On the foundations of texture: A transformational approach, Univ. of Maryland, Tech. Rept. TR-331. College Park, MD.