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Application of Population Balance Model to the Comminution of Complex Sulfide Ores

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ABSTRACT

The breakage characteristics of complex sulfide ores in a laboratory ball mill have been studied in terms of breakage rate (S) and breakage distribution (B) functions. Both of these parameters have been determined experimentally and compared with those determined by using the ESTIMILL computer program. It has been found that the ore composition and textures have marked effects on the breakage rates. The study also showed that, unlike the cases for grinding homogeneous materials, the experimental S values do not follow the power law and the B values are not generally normalizable with respect to feed size. The difficulties observed in simulating the breakage behavior of complex sulfide ores may be attributed to the fact that the ore composition changes with changing particle size. Despite these problems, the ESTIMILL program has been successful in simulating the batch grinding of complex sulfide ores using a 4-inch mill.

INTRODUCTION

Significant progress has been made in recent years in developing phenomeno-logical grinding models derived from population balance considerations. This approach has led to the development of many simulators that can be used for controlling industrial grinding circuits. Herbst and Fuerstenau (1980) have reviewed the advantages of this approach, and others (Austin, Shoji and Luckie, 1976; Siddique, 1977; Kinneberg and Herbst, 1984) have discussed various scale-up models.

In this approach, the breakage process is described by two sets of physically interpretable parameters, i.e., breakage rate and breakage distribution functions. These parameters, which are size-discretized for mathematical convenience, allow the behavior of each size fraction in the mill to be represented mathematically. However, these parameters reported in literature have been determined from experiments conducted using relatively pure and homogeneous materials, such as quartz, limestone, dolomite, coal, etc., and the breakage behavior of complex ores is not well understood.

For this reason, laboratory grinding experiments have been carried out in the present work on several different complex sulfide ores to determine the rate and the breakage distribution functions. The experimental values have been compared to those determined using the ESTIMILL program developed by Herbst et al. (1977, 1984). Using the experimentally-determined parameters as the

initial estimates, the breakage of complex ores has been simulated using the program.

MODEL

The basic batch grinding equation in the size-discretized and time-continuous form (Reid, 1965; Austin, 1971-72) is given by:

$$\frac{dm_i(t)}{dt} = \sum_{\substack{j=1 \\ i>j}}^{i-1} b_{ij}S_jm_j(t) - S_im_i(t) \tag{1}$$

where $m_i(t)$ is the fraction of total mass present in size i at time t, S_i is the fractional rate of breakage of size i, and b_{ij} is the fraction of the primary breakage products of size j which appears in the smaller size i. In the present work, the following functional forms have been used for the selection (S_i) and the cumulative breakage distribution (B_{ij}) functions, as described in the ESTIMILL program:

$$S_i = S_1 \exp \left(- \sum_{j=1}^{J} \zeta_j[\ln(\sqrt{x_i x_{i+1}}/\sqrt{x_1 x_2})]^j \right) \tag{2}$$

$$B_{ij} = \alpha_1(x_i/x_{j+1})^{\alpha_2} + (1 - \alpha_1)(x_i/x_{j+1})^{\alpha_3} \tag{3}$$

in which ζ_j, α_1, α_2 and α_3 are adjustable parameters, x refers to particle size and J represents the order of the breakage rate equation. The details of the parameter estimation scheme for α_1, α_2, α_3, S_1, ζ_1 and ζ_2 have been described by Rajamani and Herbst (1984).

EXPERIMENTAL

Five different samples of significantly different textures and mineralogical compositions were taken from the Copper Hill Mine, Ducktown District, Tennessee, and used for batch grinding experiments. Table I shows the mineralogical contents of the ore samples as determined by examining the -65 +100 mesh fractions. As shown, the ore samples are massive sulfides containing 20-65% pyrrhotite and 10-30% pyrite, with the remainder being comprised of quartz, amphibole, mica, etc. For comparison, grinding experiments have also been carried out using pure samples of quartz, pyrite and pyrrhotite, which are the major constituents of the complex ores tested.

In each grinding experiment, 100 grams of mono-sized samples were dry-ground in a 4-inch laboratory ball mill. To determine the breakage rates, the experiments were conducted at time intervals of 1, 3, 5 and 7 minutes. The media was charged with 10 lbs of steel balls (50% volume filling), which consisted of 3/8-, 1/2- and 3/4-inch diameter balls.

Some experiments were carried out using a 10-inch ball mill. The samples were wet-ground using 1000-gram samples using tap water and 22 lbs of steel balls (3/8- to 3/4-inch diameter). The ground product was dewatered using a vacuum filter and then dried in an oven at about 55-60°C. A representative sample was then removed and screened.

TABLE 1. Mineralogical Characterization of Ducktown Ore Samples (Based on -65 +100 Mesh Samples)

Ore Samples	Pyrite	Pyrrho-tite	Chalco-pyrite	Sphale-rite	Galena	Magne-tite	Gangue
Ducktown 1 - massive pyrrhotite with pyrite porphyro-blasts	28.7	45.3	3.9	0.5	0.0	4.7	17.5
Ducktown 2 - pyritic	36.3	32.8	4.2	2.4	0.1	5.3	18.7
Ducktown 3 - schistose	2.6	19.3	1.8	0.5	0.1	1.0	69.7*
Ducktown 4 - quartz - rich	8.6	19.8	3.9	0.7	0.3	8.2	58.6
Ducktown 5 - massive pyrrhotite	1.4	61.1	6.6	4.6	0.5	1.7	24.2

*mostly amphibole-biotite-garnet schist and little quartz

RESULTS

Figure 1 shows the first-order plots for dry-grinding of various Ducktown ore samples (-20 +28 mesh feed) in a 4-inch mill. It shows that most of the ore samples tested exhibit the normal first-order breakage behavior that has been observed by many other investigators (Herbst and Fuerstenau, 1968; Austin and Bhatia, 1971; Berube et al., 1979) with homogeneous materials. Similar results were obtained for the other feed sizes (i.e., -35 +48, -48 +65, -65 +100 mesh) of these Ducktown ore samples. However, the DT-3 sample does not show the first-order breakage behavior. The breakage of this ore begins to deviate from the first-order behavior after about 70% of the original feed material has been broken, which may indicate the build-up of harder materials in the coarser sizes as grinding proceeds. Most of the gangue in the DT-3 consists of amphibole-biotite-garnet schist which splits readily along parallel planes. It appears that the breakage of platelets becomes increasingly difficult with decreasing particle size.

Figure 1 also shows that the two pyrrhotite-rich ores (DT-1 and DT-2) show the fastest breakage rates, followed by quartz-rich (DT-4) and pyrite-rich (DT-2) ores. Table 2 compares these results with those of the grinding experiments conducted with relatively pure samples (-20 +28 mesh) of pyrrhotite, pyrite and quartz. The breakage rates of the pure minerals have been obtained from the first-order plots shown in Figure 2. As shown, the breakage rates of pure minerals are in the order of pyrrhotite, pyrite and quartz. However, with complex ore samples, the quartz-rich ore breaks faster than the pyrite-rich ore. This may be explained by the fact that the quartz present in the DT-4 ore is disseminated in the pyrrhotite matrix. On the other hand, the DT-2 ore contains well-crystallized pyrite grains distributed in the pyrrhotite matrix.
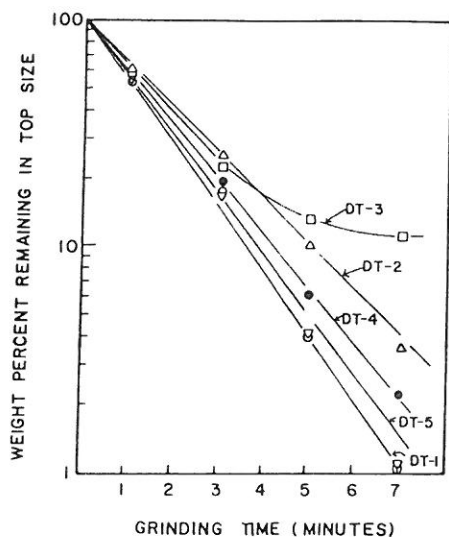
FIGURE 1. First-order plots for the dry-grinding of various Ducktown ore samples in the 4-inch laboratory ball mill using -20 +28 mesh feed.

Figure 3 compares the results of the dry- and wet-grinding of DT-1 ore in the 4-inch mill. The rate of breakage for wet-grinding of the DT-1 ore is initially higher than for dry-grinding, but then approaches a constant value in the later stages of grinding, after approximately 70% of the original feed has been broken. Several other investigators (Berube et al., 1979; Siddique, 1977) have also shown that the wet-grinding of homogeneous mineral samples, such as quartzite and limestone, exhibits a significant deviation from the first-order behavior.

Figure 4 shows the experimental and simulated breakage distribution functions for dry- and wet-grinding of the DT-1 ore sample (-20 +28 mesh feed). It can be seen that the experimental B_{ij} values for the finer sizes are slightly higher than those determined by computer simulation using the ESTIMILL program. It can also be seen that the B_{ij} values of wet-grinding are higher than those of dry-grinding. It may be noteworthy that the difference in the B_{ij} values for wet- and dry-grinding is much smaller than the difference in

TABLE 2. Comparison of First-Order Breakage Rates (min^{-1}) for Pure Minerals and Complex Sulfide Ores (-20 +28 Mesh Feed)

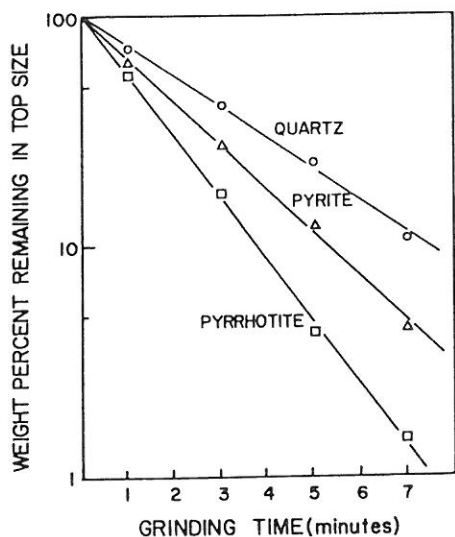| Pure Minerals			Complex Sulfide Ores		
Pyrrhotite	Pyrite	Quartz	Pyrrhotite-Rich (DT-1)	Pyrite-Rich (DT-2)	Quartz-Rich (DT-4)
0.632	0.434	0.311	0.644	0.474	0.545

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FIGURE 2. First-order plots for the dry-grinding of quartz, pyrite and pyrrhotite in the 4-inch laboratory ball mill using -20 +28 mesh feed.
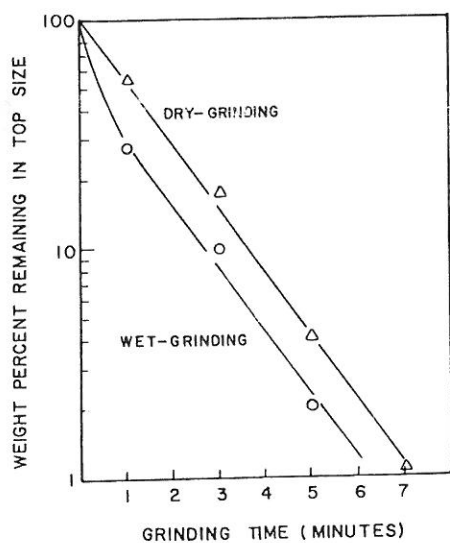

FIGURE 3. First-order plots for the dry- and wet-grinding of DT-1 ore in the 4-inch laboratory ball mill using -20 +28 mesh feed.
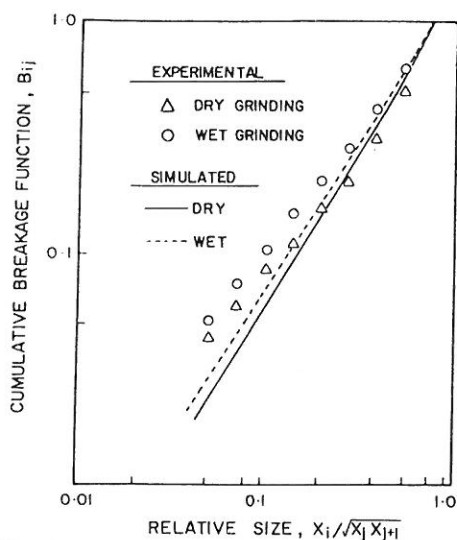
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FIGURE 4. Comparison of the experimental cumulative breakage functions and the computer-simulated values for the dry- and wet-grinding of the DT-1 ore (-20 +28 mesh feed).

the S_i values (Figure 3). This confirms the findings of several other investigators (Austin and Bhatia, 1971; Siddique, 1977) who found that the breakage distribution functions do not change significantly with changes in the mill environment. It appears, therefore, that the B_{ij} values are more characteristic of the material being ground than the grinding environment.

Many investigators (Herbst and Fuerstenau, 1968; Austin et al., 1981) have reported that the breakage distribution function is normalizable with respect to feed size in the case of dolomite, quartz, anthracite, etc. However, this has not been found to be the case for the complex ores studied in the present work. The results obtained from dry-grinding the DT-1 and DT-3 ores in a 4-inch mill show that the breakage distribution functions are not normalizable, as shown in Figures 5 and 6. The degree of normalizability is more significant with the DT-3 ore which contains a large amount of schistose minerals.

DISCUSSION

Breakage Characteristics of Complex Sulfide Ores

It has been shown in the present work that the breakage rates of complex sulfide ores are markedly influenced by the ore composition and texture. Most of the ores tested, i.e., DT-1, DT-2, DT-4 and DT-5, exhibit first order breakage behavior; however, the one that contains a large amount (59%) of amphibole-biotite-garnet schist (DT-3) tends to break in layers. Another interesting observation is that while the breakage rates of pure minerals are in the decreasing order of pyrrhotite, pyrite and quartz, those of complex ores are in the order of pyrrhotite-rich (DT-1, DT-5), quartz-rich (DT-4) and pyrite-rich (DT-2) ore. In other words, the DT-4 ore containing the hard-to-break quartz, breaks faster than the DT-2 ore containing the softer pyrite. A
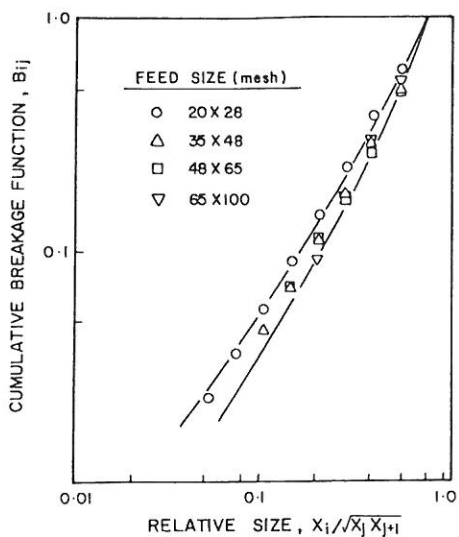
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FIGURE 5. Cumulative breakage distribution function versus normalized size for the DT-1 ore samples.
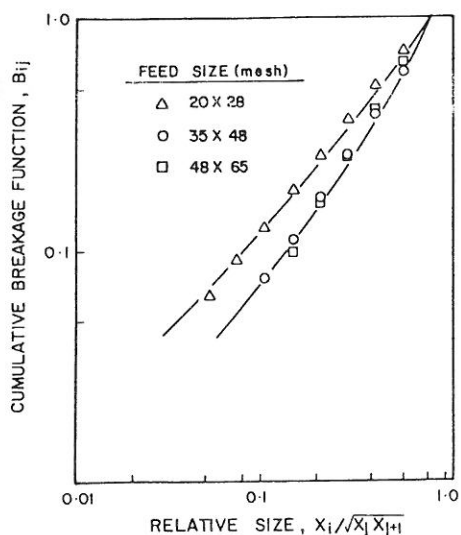

FIGURE 6. Cumulative breakage distribution function versus normalized size for the DT-3 ore samples.
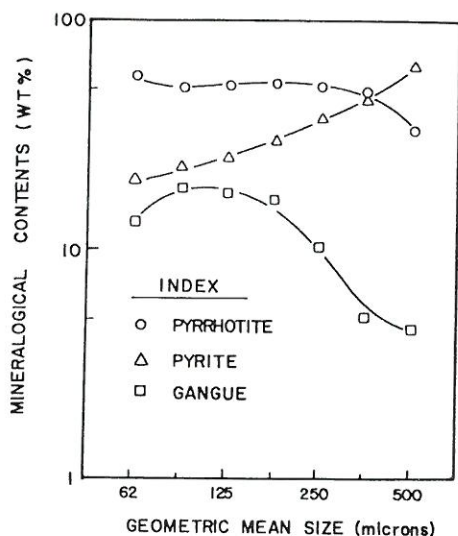
FIGURE 7. Changes in mineralogical content as a function of particle size for grinding the DT-1 ore.

possible explanation for this reversed order of breakage is that the pyrite grains in the pyrrhotite matrix are coarse-grained while the quartz grains are finely disseminated, primarily at the interstices of the larger pyrite and pyrrhotite grains of the ore (Craig et al., 1984).

Figure 7 shows the mineralogical compositions of the DT-1 ore as a function of particle size after grinding the ore for five minutes. It shows that the amount of gangue material which is largely composed of quartz increases with decreasing particle size, while the pyrite content remains high at coarser sizes. This is an indication that the finely disseminated quartz grains break out more readily than the coarse-grained pyrite despite the fact that the latter mineral is easier to break than the former when they are broken separately as homogeneous materials (Figure 2).

Many investigators who have worked on homogeneous materials showed that the selection function, $S(x)$, is a power function and can be expressed as $S(x) = ax^{\alpha}$, where x is the particle size and a and α are constants (Herbst and Fuerstenau, 1968; Austin and Bhatia, 1971). To verify this relationship, the experimental S values for DT-1 ore have been plotted as a function of x in Figure 8. Also shown in Figure 8 is the computer-simulated S_i values as obtained using the ESTIMILL program. As shown, the experimental values deviate from the linear relationship predicted from simulation. One can see, however, that the experimental S_i values exhibit linear dependency when finer size feed is used. It might be possible that by increasing the media size to feed size ratio, the linear relationship may be extended to coarser feeds.

Another interesting observation made in the present work is that the B functions of complex sulfide ores are non-normalizable. Apparently, the degree of non-normalizability depends on the complexity of the ore. A very complex ore, such as DT-3, shows a higher degree of non-normalizability (Figure 6) than the DT-1 ore (Figure 5) which consists predominantly of pyrrhotite. The DT-3 ore contains large amounts of mica schist that tends to break in layers. Furthermore, the ore composition changes with particle size, as shown
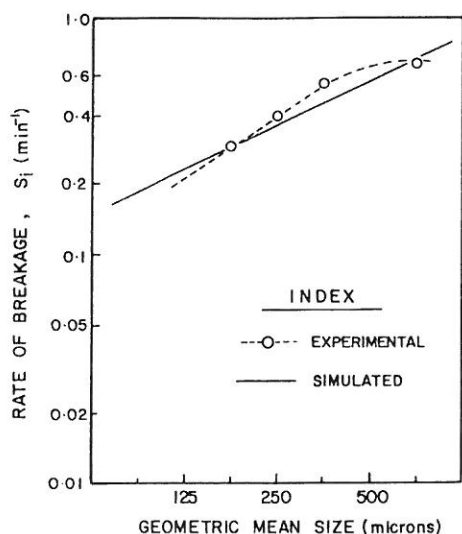
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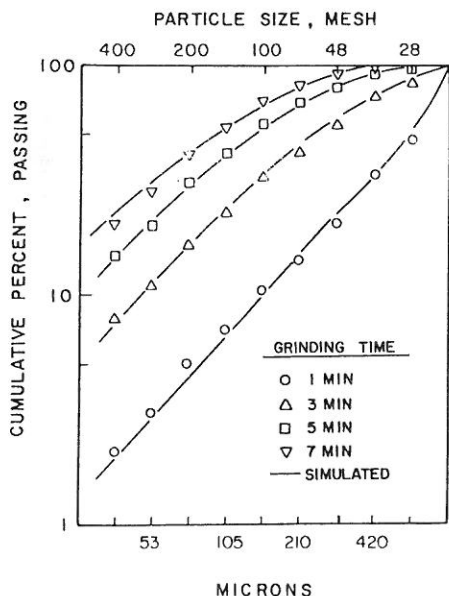FIGURE 8. Breakage rates of the DT-1 ore as a function of feed size.

FIGURE 9. Comparison between the experimental and the computer-simulated size distributions for the dry-grinding of the mono-sized (-20 +28 mesh) DT-1 ore in a 4-inch laboratory ball mill.

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in Figure 7, which will make it difficult to normalize the breakage functions.

Simulation of Batch Grinding of Complex Ores

Using the experimentally-determined selection function (S_1) and various other parameters, such as ζ_1, ζ_2, α_1, α_2 and α_3, computer simulations were carried out to predict the product size distributions in the batch ball mill grinding of mono-size feeds. Figures 9 and 10 show the results obtained from dry-grinding the DT-1 and DT-3 ores, respectively. There is a general agreement between the experimental and simulated size distributions when the ore samples are ground in a 4-inch mill. The good agreement demonstrates that the first-order assumption is valid even for the grinding of complex sulfide ores when a small laboratory ball mill is used. In the case of the troublesome DT-3 ore (Figure 10), however, the computer-predicted values fall slightly above the experimentally-determined values in the fine size range. This discrepancy is most noticeable when a short grinding time is employed. The difficulty with DT-3 ore is due to the fact that, in using the ESTIMILL program, the first-order assumption has been employed although the DT-3 ore deviates significantly from the first-order behavior (Figure 1).
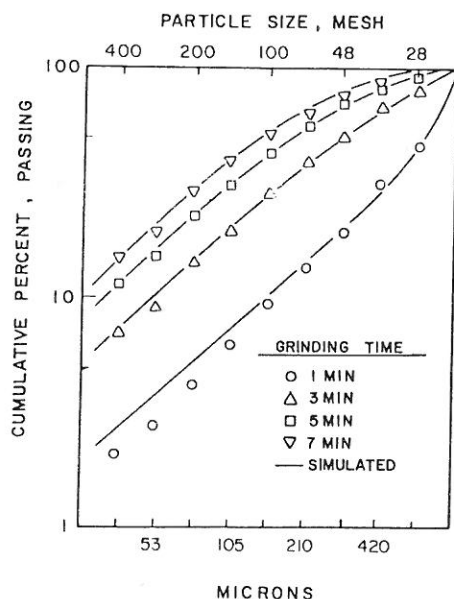

FIGURE 10. Comparison between the experimental and the computer-simulated size distributions for the dry-grinding of the mono-sized (-20 +28 mesh) DT-3 ore in a 4-inch laboratory ball mill.
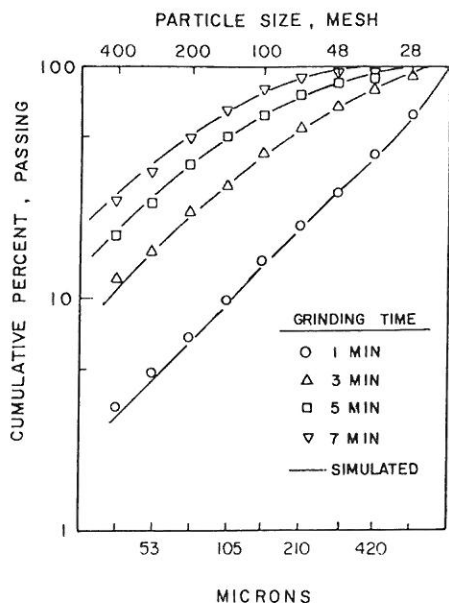
FIGURE 11. Comparison between the experimental and the computer-simulated size distributions for the wet-grinding of the mono-sized (-20 +28 mesh) DT-1 ore in a 4-inch laboratory ball mill.

Figure 11 shows the results of simulation for the wet-grinding of the DT-1 ore using the 4-inch mill. Despite the fact that wet-grinding of a complex sulfide ore deviates considerably from the first-order breakage behavior in the initial stages of grinding (Figure 3), the simulation results are in good agreement with the experimental data. The good agreement can be attributed to the fact that the breakage rate becomes linear as the grinding progresses and also that the slope of the linear portion of the curve has been used for simulation.

The S and B values obtained from mono-size feed grinding data (-20 +28 mesh) were used as initial estimates in the simulation of dry-grinding of multi-size feed (-20 mesh) in a 4-inch laboratory ball mill. Although it has been shown that the B functions are not normalizable for grinding complex ores (Figure 5 and 6), the B values were assumed to be normalizable. The results of the simulation for dry-grinding are shown in Figure 12, which shows that there is good agreement between the experimental and simulated results. Although not shown in this paper, equally good agreement has been obtained for the wet-grinding of multi-size feed in a 4-inch mill (Choi, 1982). This agreement suggests that the normalizability of B functions is not as critical as the S functions in simulation.

Figure 13 shows the simulation results obtained for the wet-grinding of DT-1 (-20 mesh) in a 10-inch mill. Due to the lack of experimental S_i and B_{ij} values obtained with this 10-inch mill, the initial estimates have been made as suggested in the ESTIMILL program. As shown, the simulations are not as good as those obtained with the 4-inch mill. The simulation results become increasingly poor with increasing grinding time. Even after a short grinding
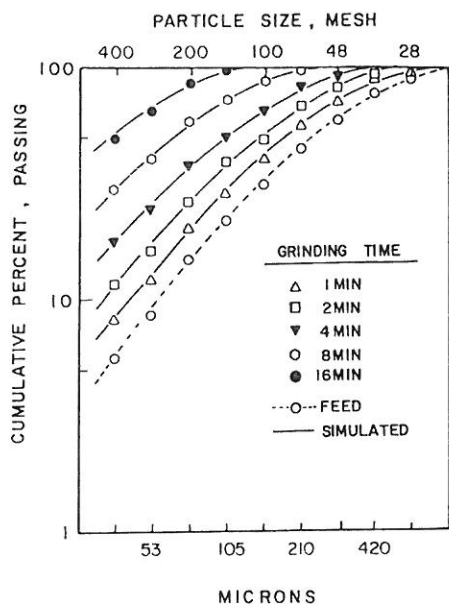
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FIGURE 12. Comparison between the experimental and the computer-simulated size distributions for the dry-grinding of multi-sized (-20 mesh) DT-1 ore in a 4-inch laboratory ball mill.
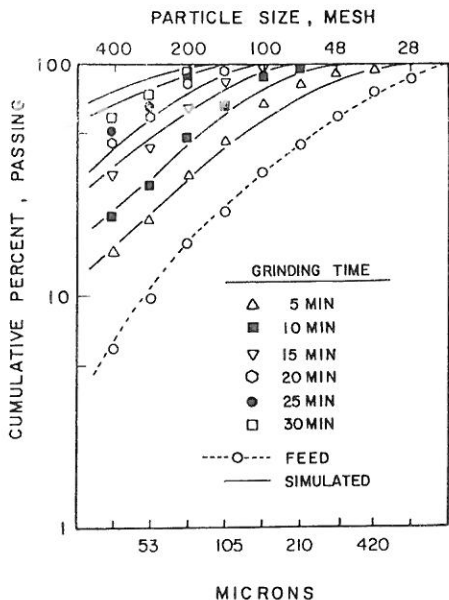

FIGURE 13. Comparison between the experimental and the computer-simulated size distributions for the wet-grinding of multi-sized (-20 mesh) DT-1 ore in a 10-inch laboratory ball mill.

time, the simulated results deviate considerably at coarser particle size ranges. Attempts have been made to improve the simulation using S_i values obtained by using the second-order and the third-order terms of Eq. [2], but without much success. The reason for this difficulty may be traced back to the fact that the functional forms for the S_i and B_{ij} parameters used in the ESTIMILL program have been derived on the basis of experimental data obtained with homogeneous materials. Gupta et al. (1981), who used the back-calculation method for simulating a pyrite-rich complex ore from New Brunswick, showed that a more general and flexible functional form needs to be used to estimate B_{ij} of a complex ore. They have also shown that S_i values for their complex ores are best described when a fourth-order polynomial form of Eq. [2] is used.

SUMMARY AND CONCLUSIONS

The breakage characteristics of several complex sulfide ores during laboratory ball milling have been studied in terms of breakage rate and breakage distribution parameters. It has been found that the ore composition and textures have marked effects on the breakage rates. Most of the complex sulfide ores tested (DT-1, DT-2, DT-4 and DT-5) exhibit first-order breakage behavior, except the one (DT-3) containing large amounts of schistose gangue minerals. The study also showed that, unlike the cases for grinding homogeneous materials, the experimental S_i values do not follow the power law and the B_{ij} values are not generally normalizable with respect to feed size. It has been found that the degree of non-normalizability depends on the complexity of the ore.

Despite these problems in grinding complex ores, the results of computer simulation using the ESTIMILL program are in reasonable agreement with the experimental results obtained using the 4-inch mill. However, when a 10-inch mill is used for grinding complex sulfide ores for longer grinding times, the simulation does not predict the size distribution accurately, which may be attributed to the classification that may occur during grinding.

ACKNOWLEDGMENTS

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