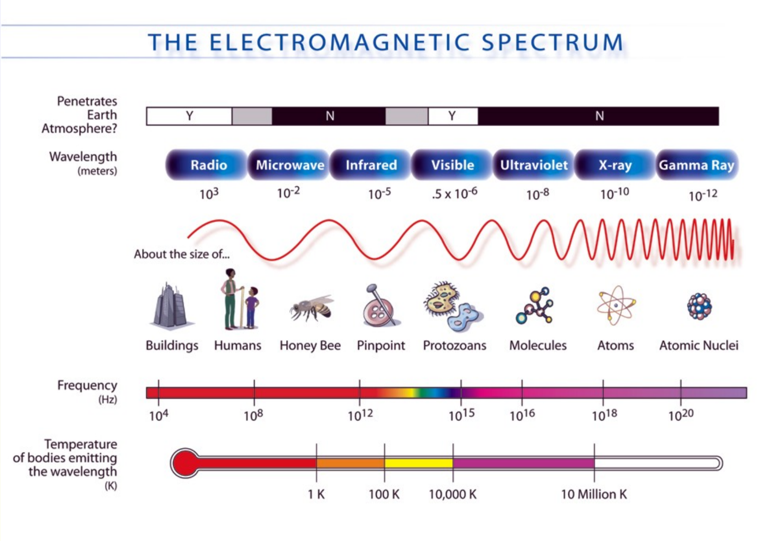
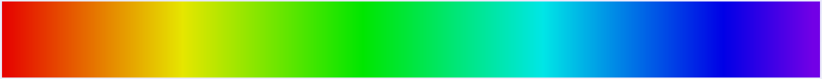
The Electromagnetic Spectrum



Color



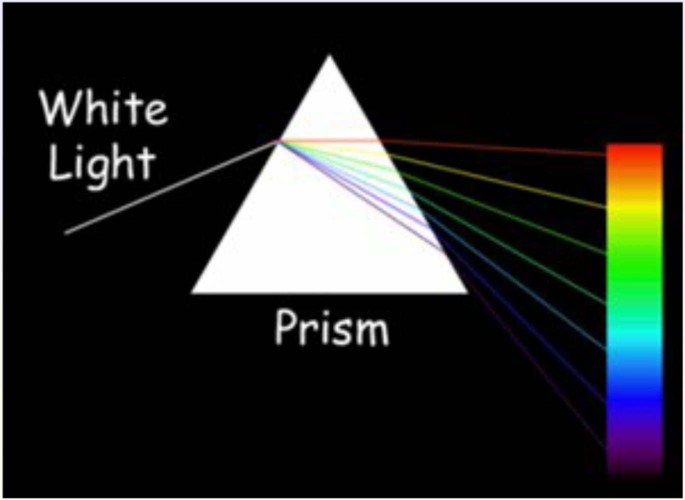
Definition

Color is a visual sensation related to the wavelength distribution of the light energy hitting the retina of the eye.

When that wavelength distribution is dominated by wavelengths centered in the

- 425nm band, the eye sees violet
- 470nm band, the eye sees blue
- 525nm band, the eye sees green
- 575nm band, the eye sees yellow
- 665nm band, the eye sees red

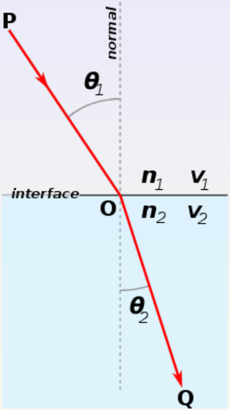
The Prism



Most of the scientists in Newton's day thought that the prisms were merely adding the color to the light, rather than separating the colors from the light.

Refraction

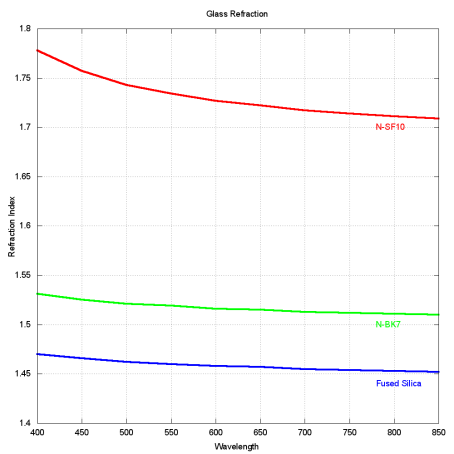
The speed of light in a medium changes with the wavelength of the light.



$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

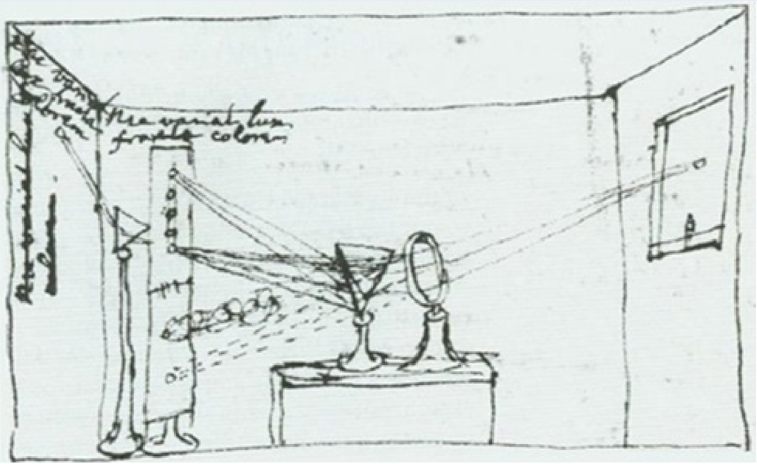
Denser mediums bend light toward the normal.

Refractive Index As a Function of Wavelength

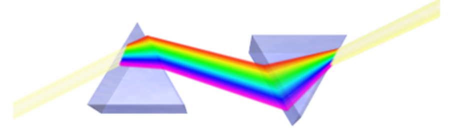


Refractive Index As a Function of Wavelength for Different Kinds of Glass

Newton’s Two Prism Experiments



Newton’s Two Prism Experiments

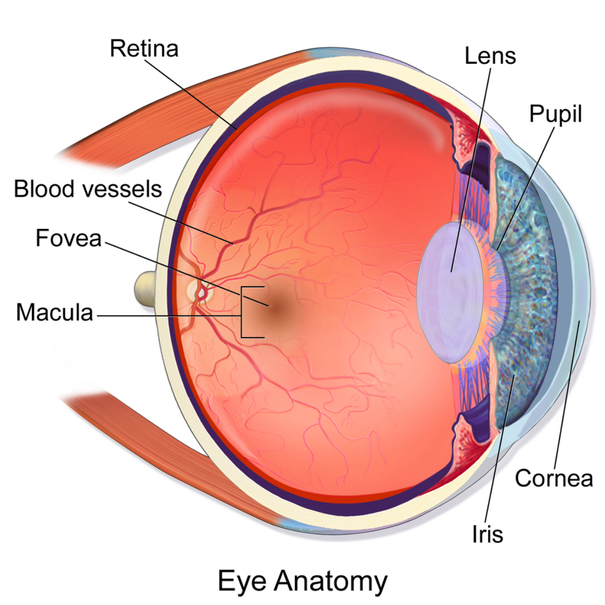


Newton was the first one to prove that white light is made of all the colors of the rainbow. The proof:
He placed a second prism upside-down in front of the first prism. The band of colors that was split up by the first prism was combined again by the second prism into white sunlight.

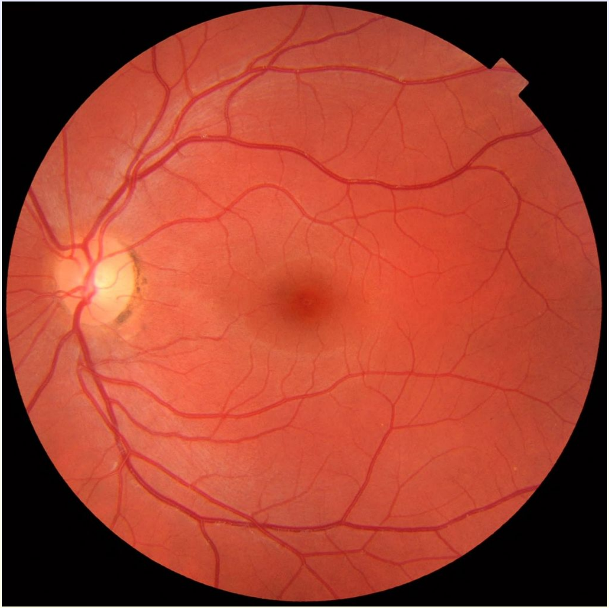
Newton’s Two Prism Experiment

Virtual Simulation

The Eye


Retina
Lens
Pupil
Blood vessels
Fovea
Macula
Cornea
Iris
Eye Anatomy

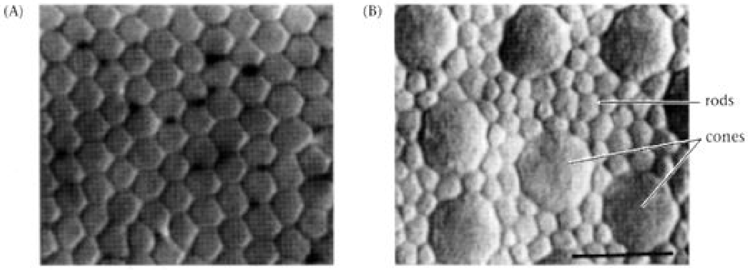
Retina

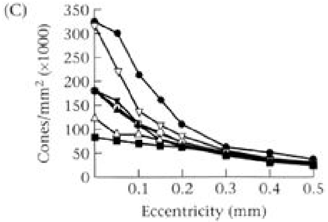


The Retina

- There are two types of photoreceptors in the human retina, rods and cones.
- Rods
  - Rods are responsible for vision at low light levels (scotopic vision)
  - Have a low spatial acuity
  - Do not participate in color visual sensation
  - 100 million rods
- Cones
  - Cones are active at higher light levels (photopic vision)
  - Are capable of color vision
  - Have a high spatial acuity.
  - Three types of cones: called S-cone, M-cone, L-cone
  - The central $300\mu m$ of the fovea is composed totally of cones
  - Fovea has $1.5^\circ - 2^\circ$ visual angle
  - 5 million cones

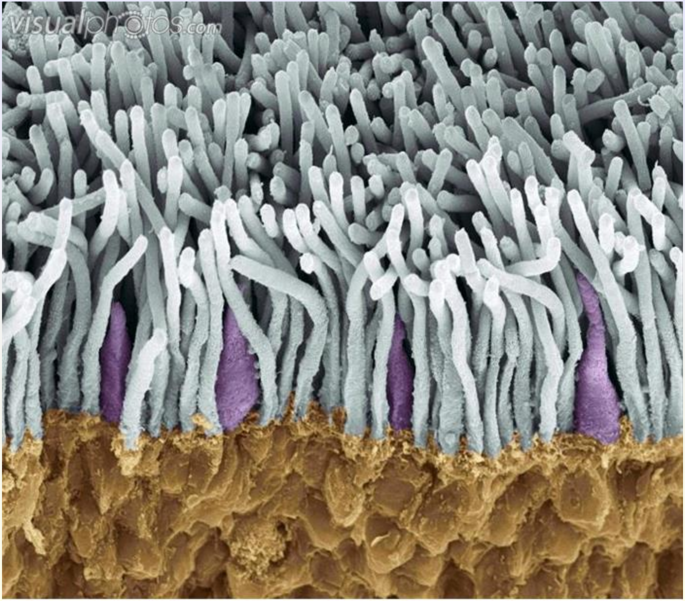
Rods and Cones



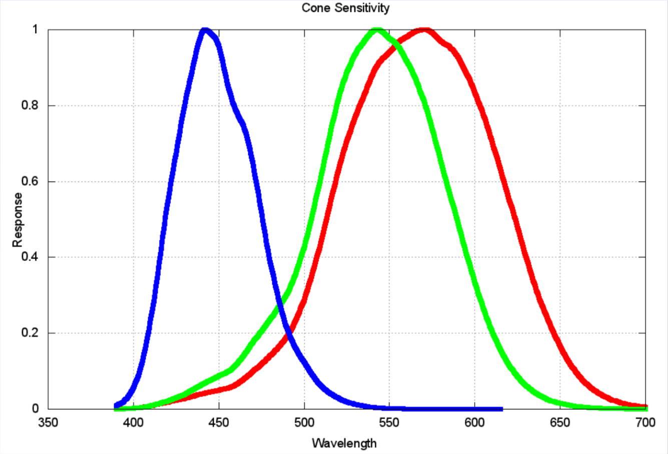


3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.

Rods and Cones

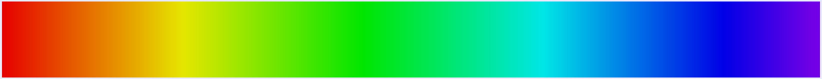


Cone Relative Sensitivity



Data from Stockman and Sharpe Colour Vision Laboratory
www.cvrl.org
Stockman and Sharpe, **Vision Research**, vol. 40, 2000,pp. 1711-1737.

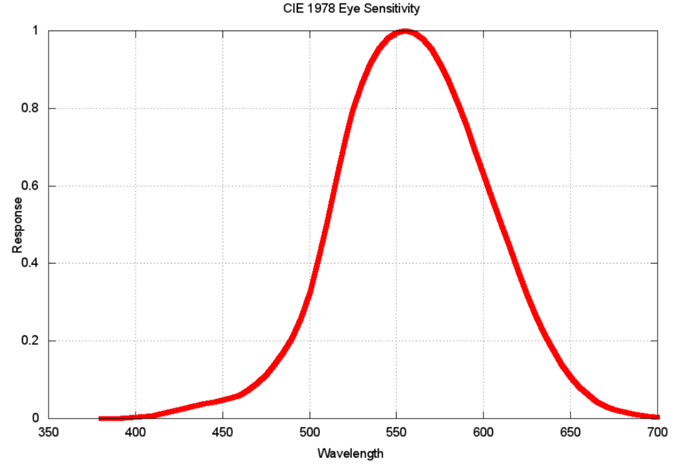
Color Response



- λ wavelength
- $p_S(\lambda) \geq 0$ cone-S sensitivity for short wavelengths
- $p_M(\lambda) \geq 0$ cone-M sensitivity for medium wavelengths
- $p_L(\lambda) \geq 0$ cone-L sensitivity for long wavelengths
- $e(\lambda) \geq 0$ spectral radiance emitted at wavelength λ

$$\begin{aligned}
\text{Response-S } r_S &= \int e(\lambda) p_S(\lambda) d\lambda \\
\text{Response-M } r_M &= \int e(\lambda) p_M(\lambda) d\lambda \\
\text{Response-L } r_L &= \int e(\lambda) p_L(\lambda) d\lambda
\end{aligned}$$

CIE 1978 Eye Sensitivity



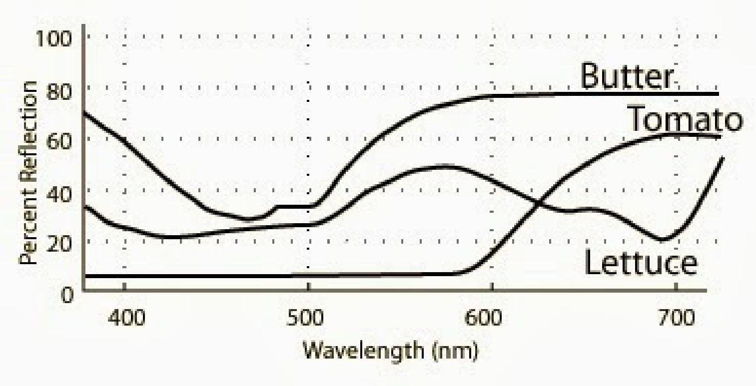
Shows the overall eye sensitivity as a function of wavelength.
The eye is much more sensitive to greens than blues or reds.

Peacock Mantis Shrimp

3-18cm; forearms can acceleration similar to .22 caliber bullet, 340 pounds force per strike. Can see from infrared through ultraviolet. Eight different types of photoreceptors. Can distinguish circularly polarized light.

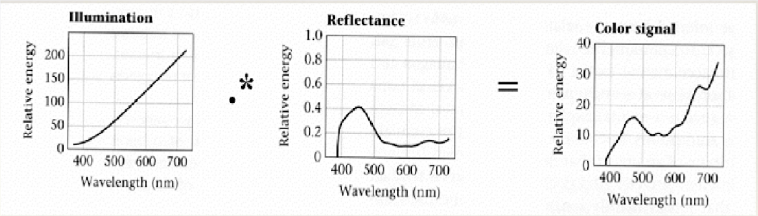
Butter, Tomatoe and Lettuce

Spectral Radiance emitted depends on the spectral radiance of the light source and the percent reflection of the illuminated surface.



Illuminance, Reflectance, Radiance

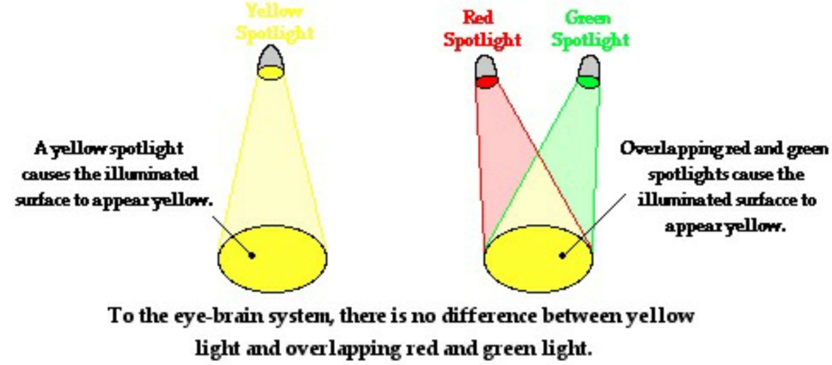
The spectral radiance is the component-wise product of the reflectance times the illuminance. Reflectance can depend on the angle of illumination.



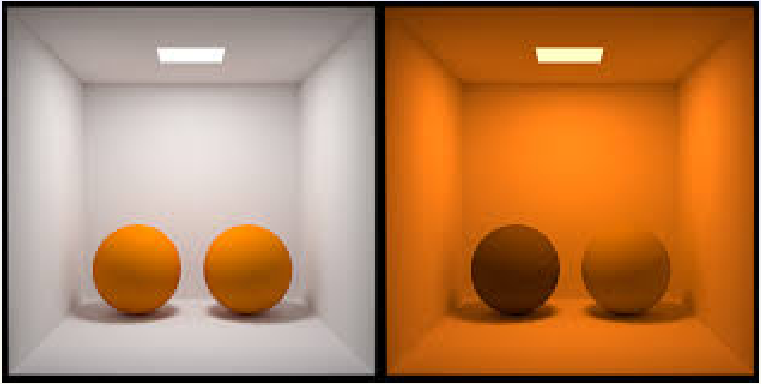
Multiple Conditions Have Appearance as Yellow

Seeing yellow can mean

- Wavelengths between $577 - 597\mu m$
- Any Balanced Mixture of Red and Green
  - 700nm and 530nm
  - infinitely many other mixtures

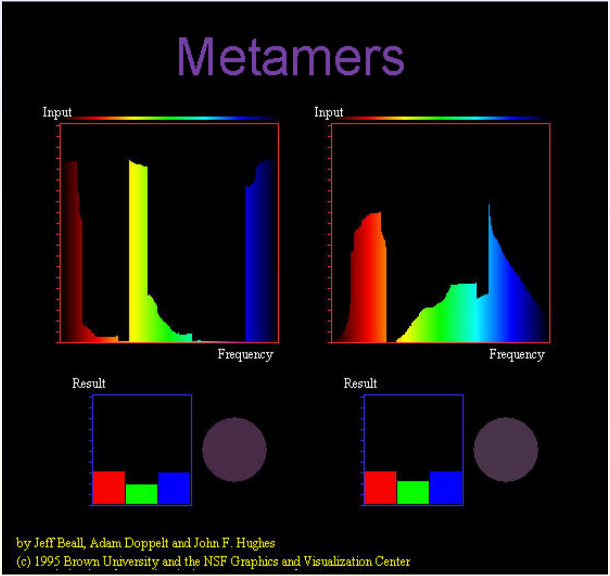


Color Metamerism



Two objects with different spectral reflectance functions can appear to be the same color in one light and appear to have different colors in another light.

Metamers


Metamers
Input
Frequency
Result
Input
Frequency
Result
by Jeff Beall, Adam Doppelt and John F. Hughes
(c) 1995 Brown University and the NSF Graphics and Visualization Center

Multiple Conditions Can Have Same Color Appearance

The appearance of color is determined by the response of the of the cones: (r_S, r_M, r_L)

$$\begin{aligned} \text{Response-S } r_S &= \int e(\lambda) p_S(\lambda) d\lambda \\ \text{Response-M } r_M &= \int e(\lambda) p_M(\lambda) d\lambda \\ \text{Response-L } r_L &= \int e(\lambda) p_L(\lambda) d\lambda \end{aligned}$$

If for two instances

$$(r_{1S}, r_{1M}, r_{1L}) = (r_{2S}, r_{2M}, r_{2L})$$

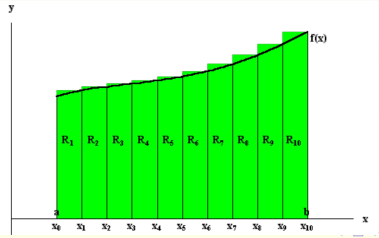
the instances have the same color appearance.

Integration

$$I = \int_a^b f(x)dx$$

Let

$$\begin{aligned} x_0 &= a \\ x_n &= a + n\Delta = x_{n-1} + \Delta \\ x_{10} &= b \\ R_n &= x_n\Delta \\ I &\approx \sum_{n=1}^{10} f(x_n)\Delta \end{aligned}$$



Convex Combinations

If two spectral radiances produce the same color appearance, then their convex combination produces that color appearance.

Suppose

$$\begin{aligned} r_S &\approx \sum_{n=1}^{N} e_1(\lambda_n) p_S(\lambda_n)\Delta \\ r_S &\approx \sum_{n=1}^{N} e_2(\lambda_n) p_S(\lambda_n)\Delta \end{aligned}$$

Let $0 \leq \alpha \leq 1$. Then

$$\begin{aligned} 0 &\leq \alpha e_1(\lambda) + (1-\alpha) e_2(\lambda) \quad \text{A Convex Combination of } e_1 \text{ and } e_2 \\ r_S' &\approx \sum_{n=1}^{N} [\alpha e_1(\lambda_n) + (1-\alpha) e_2(\lambda)] p_S(\lambda_n)\Delta \\ &= \alpha \sum_{n=1}^{N} e_1(\lambda_n) p_S(\lambda_n)\Delta + (1-\alpha) \sum_{n=1}^{N} e_2(\lambda_n) p_S(\lambda_n)\Delta \\ &= \alpha r_S + (1-\alpha) r_S = r_S \end{aligned}$$

Sum Approximations

Define

$$\begin{aligned} e(\lambda_n) &= e_n \\ p_{Sn} &= p_S(\lambda_n)\Delta \\ p_{Ln} &= p_L(\lambda_n)\Delta \\ p_{Mn} &= p_M(\lambda_n)\Delta \end{aligned}$$

Then

$$\begin{aligned} r_S &= \sum_{n=1}^{N} e_n p_{Sn} \\ r_M &= \sum_{n=1}^{N} e_n p_{Mn} \\ r_L &= \sum_{n=1}^{N} e_n p_{Ln} \\ \begin{pmatrix} r_S \\ r_M \\ r_L \end{pmatrix} &= \begin{pmatrix} p_{S1} & \dots & p_{SN} \\ p_{M1} & \dots & p_{MN} \\ p_{L1} & \dots & p_{LN} \end{pmatrix} \begin{pmatrix} e_1 \\ \vdots \\ e_N \end{pmatrix} \\ r &= Pe \end{aligned}$$

Notation Convention

Let

$$e^{N\times 1} = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}$$

Then $e^{N\times 1} \geq 0$ means $e_n \geq 0, n = 1, \ldots, N$

Convex Combinations

If two spectral radiances produce the same color appearance, then their convex combination produces that color appearance.

$$r^{3\times 1} \quad = \quad P^{3\times N} e^{n\times 1}$$

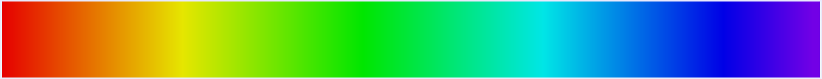
Suppose $e_1 \geq 0$ and $e_2 \geq 0$ satisfy

$$\begin{aligned} r &= Pe_1 \\ r &= Pe_2 \end{aligned}$$

Let $0 \leq \alpha \leq 1$. Then,

$$\begin{aligned} \alpha e_1 + (1-\alpha)e_2 &\geq 0 \\ P(\alpha e_1 + (1-\alpha)e_2) &= \alpha P e_1 + (1-\alpha)Pe_2 \\ &= \alpha r + (1-\alpha) r \\ &= r \end{aligned}$$

Seeing the Same Color



Let $r^{3\times 1}$ and $P^{3\times N}$ be given and let $e_0^{N\times 1} \geq 0$ be any spectral radiance vector satisfying $r = Pe_0$, $N > 3$. Consider the solution set

$$S = \{e \mid Pe = r \text{ and } e \geq 0\}$$

The set $K = \{a \mid Pa = 0\}$ is called the Kernel of P. Then,

$$S = \{e \mid e = e_0 + a,\ a \in K \text{ and } e \geq 0\}$$

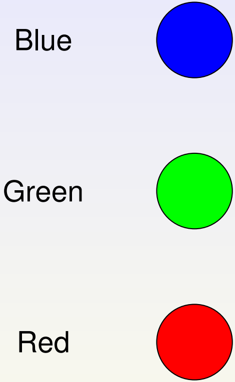
S is the positive quadrant of {the subspace K translated by e_0}
S has infinitely many elements.
There are many distinctly different spectral radiance emittances that we see as the same color.

Color Systems

There are many systems for specifying color.

- RGB
- HSB
- HSL
- CIE
- CIELa*b*
- Munsell

RGB



Can it be possible for a mixture of Red, Green, and Blue to have the appearance of any given color?

How Does RGB Make Arbitrary Colors

Let $r^{3\times 1}$ be a given cone response vector.
r corresponds to a particular color appearance.
Suppose that
Blue has spectral radiance at a single wavelength α at intensity e_α
Green has spectral radiance at a single wavelength β at intensity e_β
Red has spectral radiance at a single wavelength γ at intensity e_γ
Then the spectral radiance vector e is all 0's with the exception of components α, β, and γ

$$\begin{pmatrix} r_S \\ r_M \\ r_L \end{pmatrix} = \begin{pmatrix} p_{S\alpha} & p_{S\beta} & p_{S\gamma} \\ p_{M\alpha} & p_{M\beta} & p_{M\gamma} \\ p_{L\alpha} & p_{L\beta} & p_{L\gamma} \end{pmatrix} \begin{pmatrix} e_\alpha \\ e_\beta \\ e_\gamma \end{pmatrix}$$

$$\begin{pmatrix} e_\alpha \\ e_\beta \\ e_\gamma \end{pmatrix} = \begin{pmatrix} p_{S\alpha} & p_{S\beta} & p_{S\gamma} \\ p_{M\alpha} & p_{M\beta} & p_{M\gamma} \\ p_{L\alpha} & p_{L\beta} & p_{L\gamma} \end{pmatrix}^{-1} \begin{pmatrix} r_S \\ r_M \\ r_L \end{pmatrix} = r$$

If $(e_\alpha, e_\beta, e_\gamma) \geq 0$ then the color specified by (r_S, r_M, r_L) can be produced by a mixture of red, green and blue.

Can RGB Make An Arbitrary Color?

Let

- $e_{red}^{N \times 1}$ be the spectral radiance of a given red source that produces cone response $a^{3 \times 1} = Pe_{red}^{N \times 1}$
- $e_{green}^{N \times 1}$ be the spectral radiance of a given green source that produces cone response $b^{3 \times 1} = Pe_{green}^{N \times 1}$
- $e_{blue}^{N \times 1}$ be the spectral radiance of a given blue source that produces cone response $c^{3 \times 1} = Pe_{blue}^{N \times 1}$
- r be a cone response vector specifying a given color appearance

Does there exist mixing coefficients $\alpha, \beta, \gamma \geq 0$ such that

$$\alpha e_{red} + \beta e_{green} + \gamma e_{blue}$$

produces the color r?

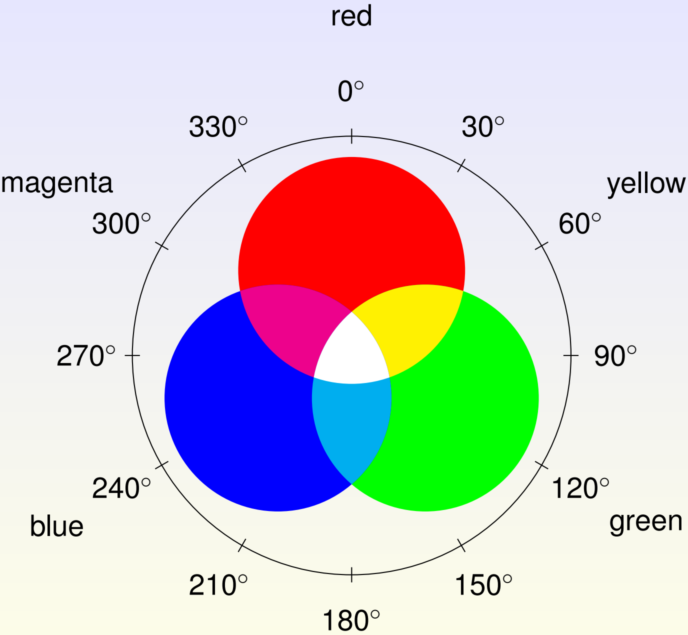
Can RGB Make An Arbitrary Color

Does there exist mixing coefficients $\alpha, \beta, \gamma \geq 0$ such that

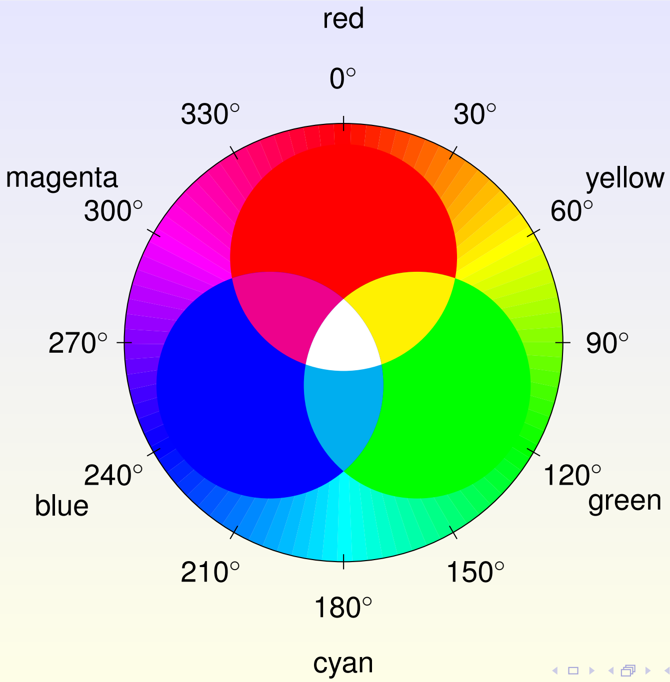
$$\alpha e_{red} + \beta e_{green} + \gamma e_{blue}$$

produces the color r?

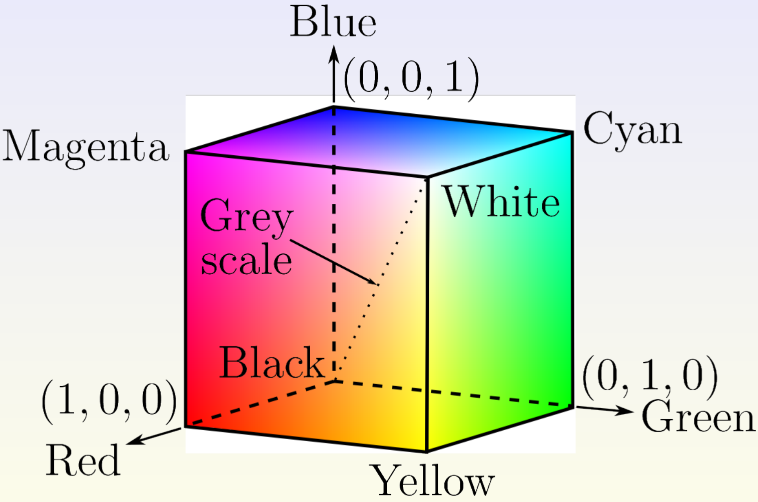
$$\begin{aligned}
r &= P(\alpha e_{red} + \beta e_{green} + \gamma e_{blue}) \\
&= \alpha P e_{red} + \beta P e_{green} + P \gamma e_{blue} \\
&= \alpha a + \beta b + \gamma c \\
&= \begin{pmatrix} | & | & | \\ a & b & c \\ | & | & | \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \\
\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} &= \begin{pmatrix} | & | & | \\ a & b & c \\ | & | & | \end{pmatrix}^{-1} r
\end{aligned}$$



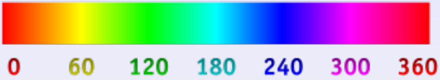
red
0°
330°
30°
magenta
yellow
300°
60°
270°
90°
240°
120°
blue
green
210°
150°
180°


red
0°
30°
yellow
60°
90°
120°
green
150°
180°
cyan
210°
blue
240°
270°
300°
magenta
330°

RGB Color Cube

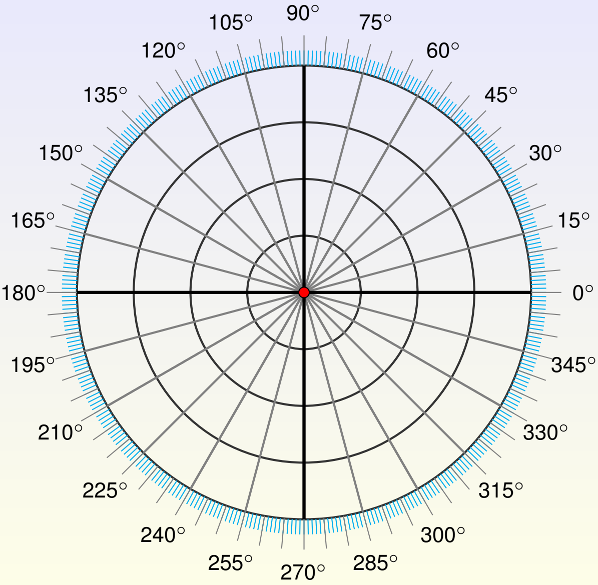


Hue



- The perceived color attribute of a visual sensation according to which an area appears to be similar to one of the perceived colors: red, yellow, green, and blue, or to some combination of two of them
- It can be measured in angular degrees counter-clockwise around the cylinder
  - • Red 0°
  - • Yellow 60°
  - • Green 120°
  - • Cyan 180°
  - • Blue 240°
  - • Magenta 300°
- The Hue specified by (R, G, B) is the same hue as specified by $\lambda(R, G, B)$ for $0 \leq \lambda \leq 1$

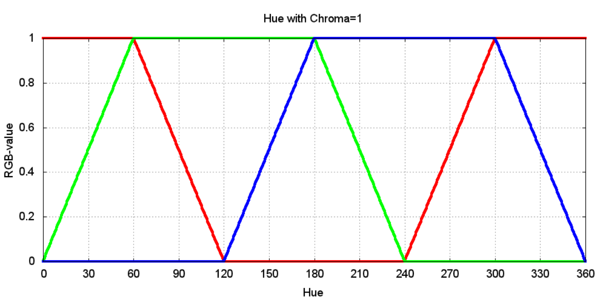
Polar Coordinate System



Hue and Chroma

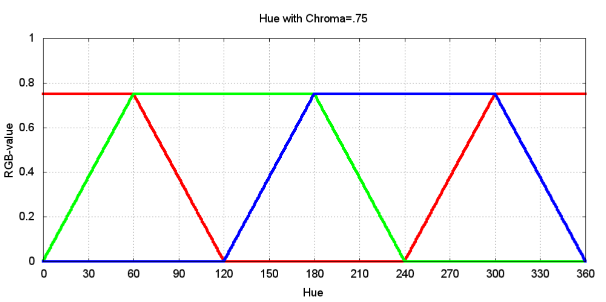
- Each color can be represented as a point on a polar (Hue,Chroma) plot
- The Hue is the angle coordinate
- The Chroma is the radial coordinate

Chroma = 1



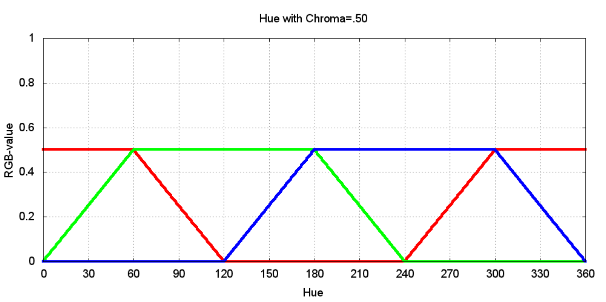
RGB values as a function of Hue with Chroma = 1.

Chroma = .75



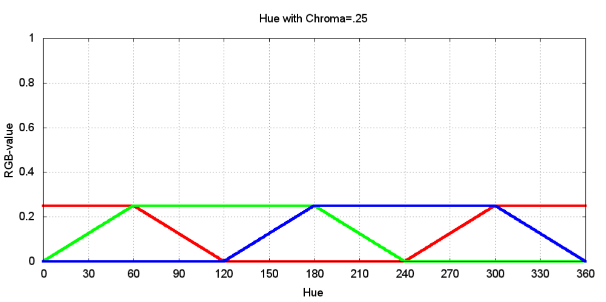
RGB values as a function of Hue with Chroma = .75

Chroma = .50



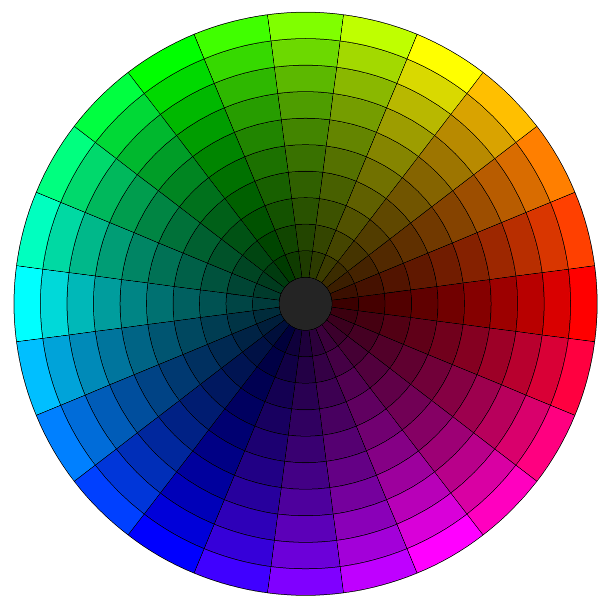
RGB values as a function of Hue with Chroma = .50

Chroma = .25

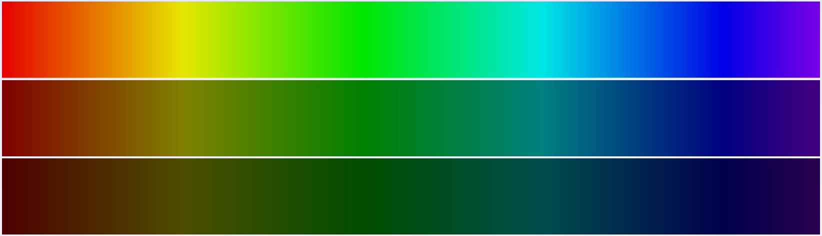


RGB values as a function of Hue with Chroma = .25

Hue Chroma in Polar Plot

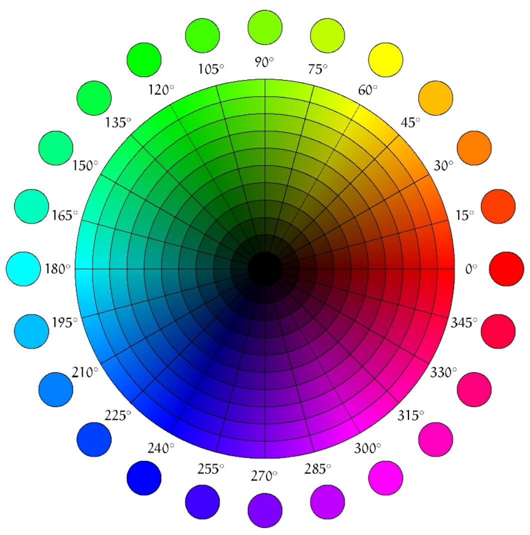


Brightness



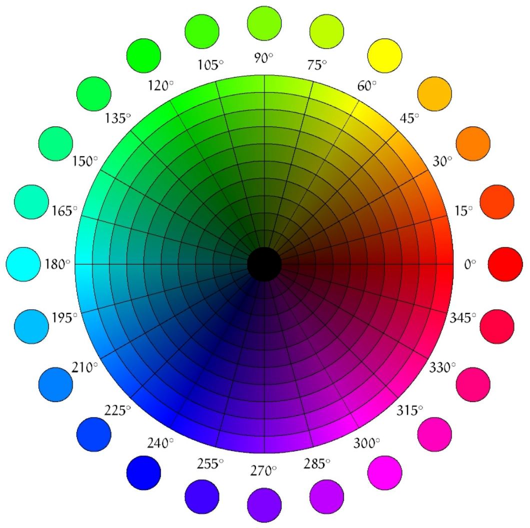
- The attribute of a visual sensation according to which an area appears to emit more or less light
- Brightness can be measured in percent from black (0) to white (100) or in $[0, 1]$
- Brightness in HSB is defined by $\max\{R, G, B\}$
- Colors with the same HSB Brightness do not have the same luminance
- At 0% brightness, hue is meaningless

Hue and Brightness



Brightness decreases by .1 as each annulus gets closer to the center.

Hue and Brightness



Brightness decreases by 15% as each annulus gets closer to the center.

Saturation

- Saturation indicates the degree to which the hue differs from a neutral gray
- Saturation values run from 0%, which is no color saturation, to 100%, which is the fullest saturation of a given hue at a given percentage of brightness
- In HSB, Saturation is defined by $\frac{\max\{R,G,B\}-\min\{R,G,B\}}{\max\{R,G,B\}}$
- At 0% saturation, hue is meaningless.
- In the above strips, Saturation goes from 1, .6 .25, .15 with brightness held at 1.
- In the bottom strips, Saturation goes from 1, .6, .25 .15 with brightness held at .5

HSB: Hue Saturation Brightness

Let R,G,B the the values of Red, Green, and Blue.
The brightness b is given by

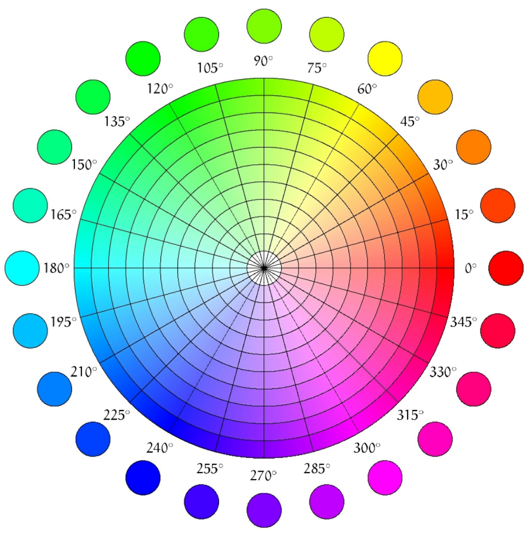
$$b = \max\{R, G, B\}$$

The saturation s is given by

$$s = \frac{\max\{R, G, B\} - \min\{R, G, B\}}{\max\{R, G, B\}}$$

The saturation of (R, G, B) is the same as the saturation of $(\lambda R, \lambda G, \lambda B)$

Saturation



The saturation is reduced by 15% as the rings move inward to the center.
The brightness is fixed at 1.

Motivation for the HSB System

Suppose a color is specified by (R, G, B).

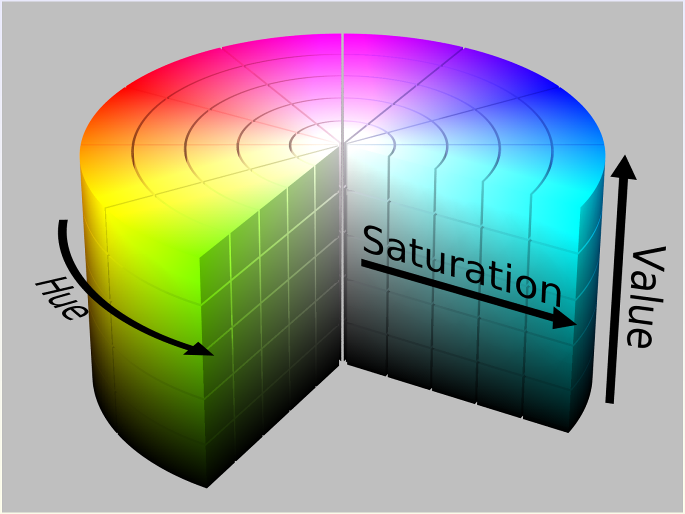
- $c_{max} = max\{R, G, B\}$
- $c_{min} = min\{R, G, B\}$
- $c_{min}(1, 1, 1)$ represents the amount of white added to the base color
- $(R, G, B) = c_{min}(1, 1, 1) + (R - c_{min}, G - c_{min}, B - c_{min})$

Suppose $c_{max} = R$ and $c_{min} = B$.

$$\begin{aligned}
(R, G, B) &= c_{min}(1, 1, 1) + (c_{max} - c_{min}, G - c_{min}, 0) \\
&= c_{min}(1, 1, 1) + (c_{max} - c_{min})(1, \frac{G - c_{min}}{c_{max} - c_{min}}, 0) \\
&= c_{min}(1, 1, 1) + c_{max}\left[\frac{c_{max} - c_{min}}{c_{max}}\right]\left(1, \frac{G - c_{min}}{c_{max} - c_{min}}, 0\right)
\end{aligned}$$

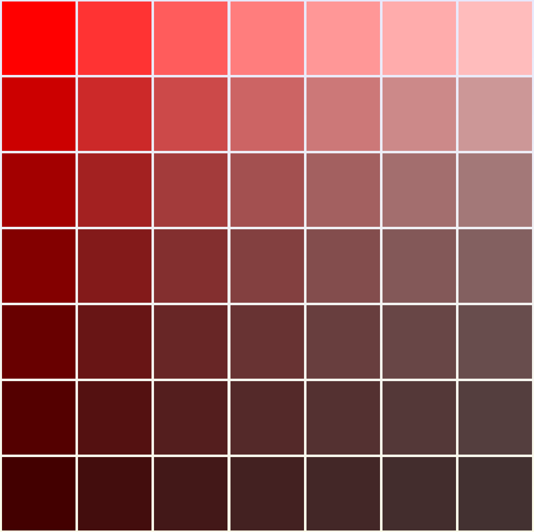
- c_{max} is the HSB Brightness
- $\frac{c_{max} - c_{min}}{c_{max}}$ is the HSB Saturation

The HSB Cylinder



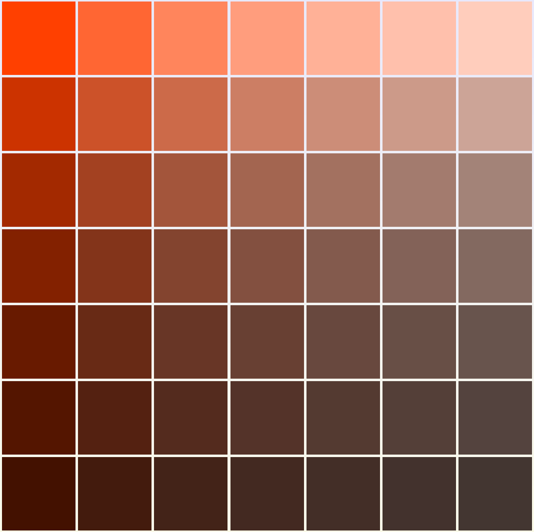
HSB: Hue Saturation Brightness is same as HSV: Hue Saturation Value

HSB Cylinder Slab



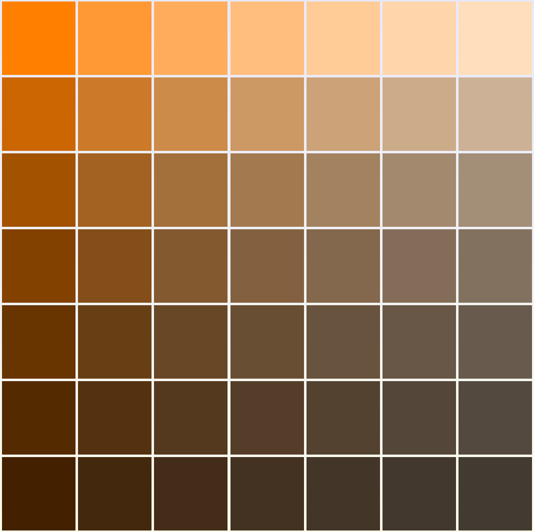
Hue: 0°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



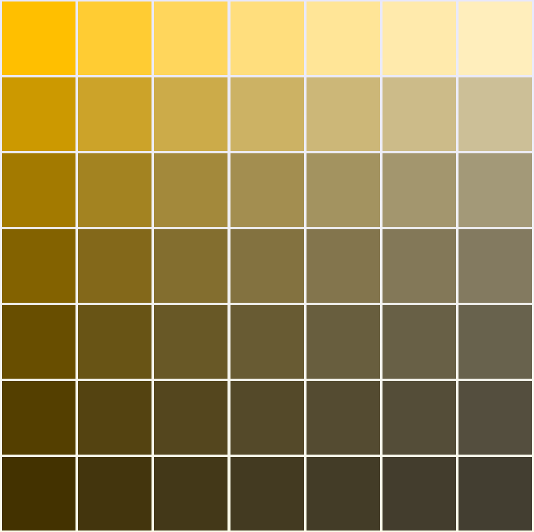
Hue: 15°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



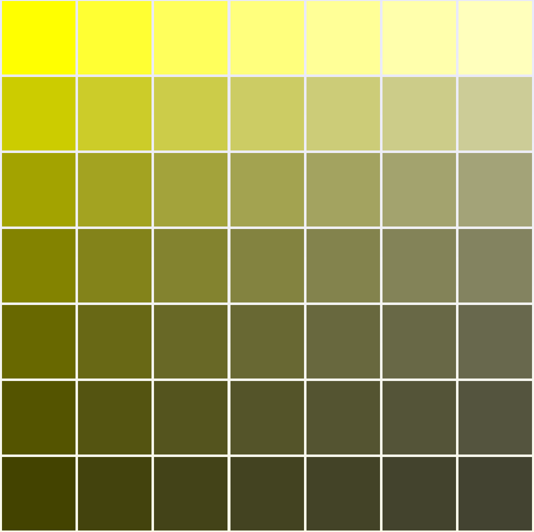
Hue: 30°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



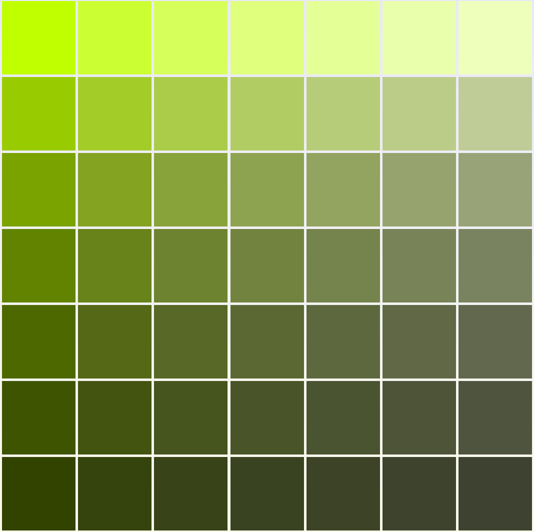
Hue: 45°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



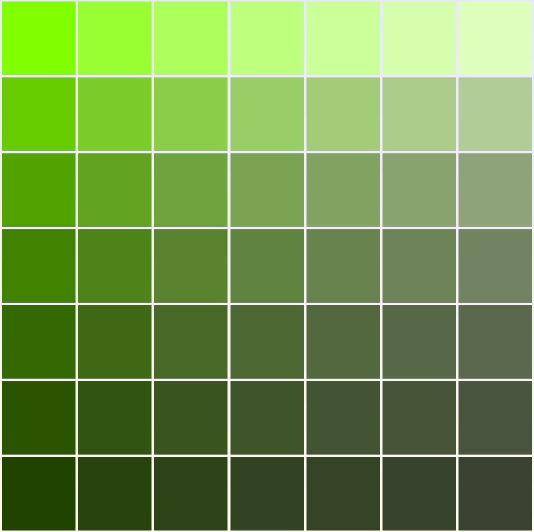
Hue: 60°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



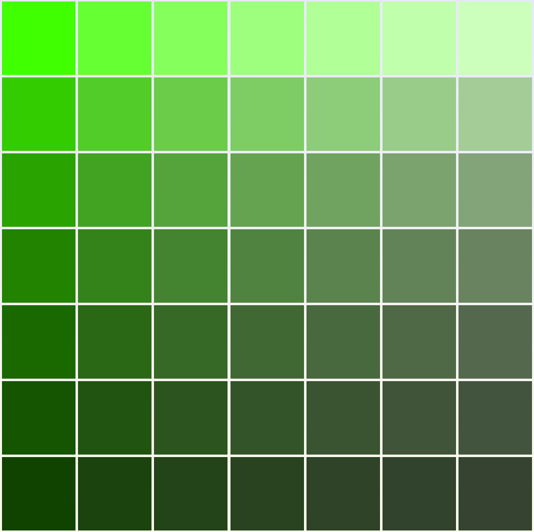
Hue: 75°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



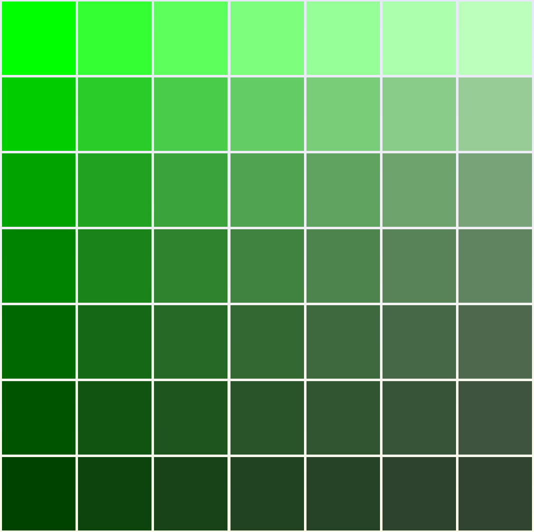
Hue: 90°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



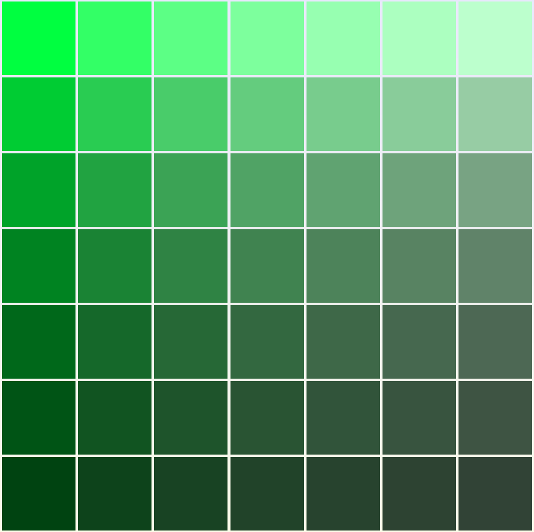
Hue: 105°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



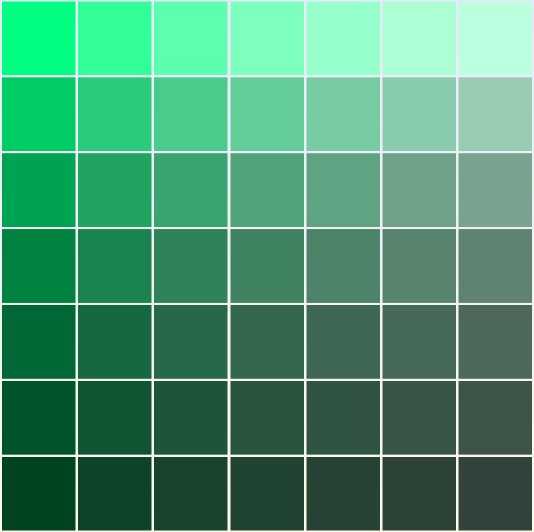
Hue: 120°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



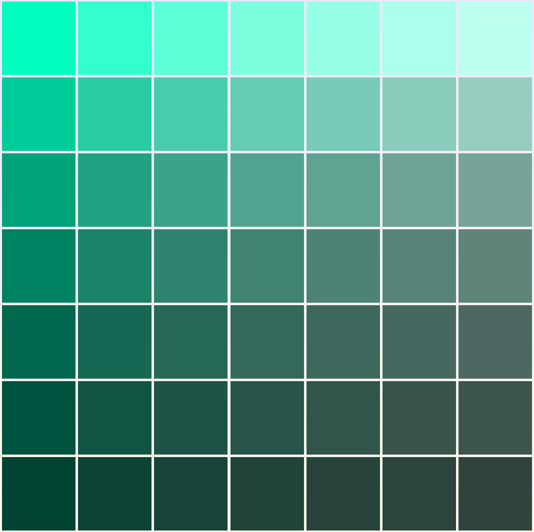
Hue: 135°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



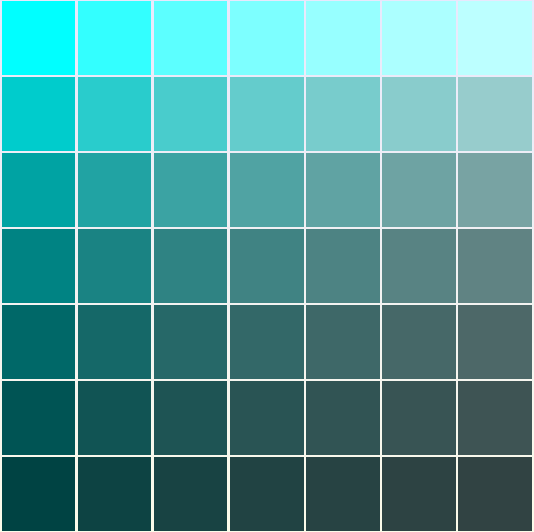
Hue: 150°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



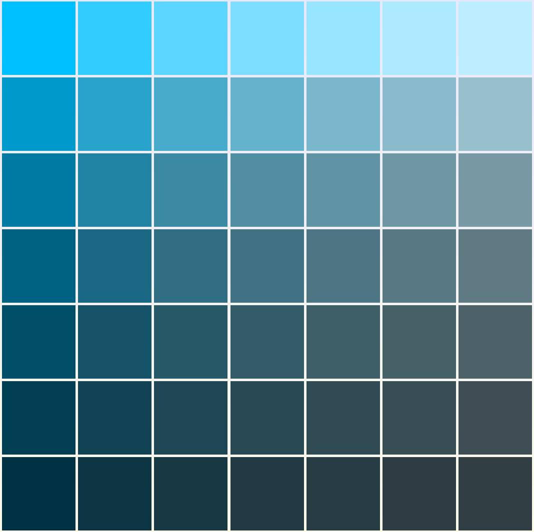
Hue: 165°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



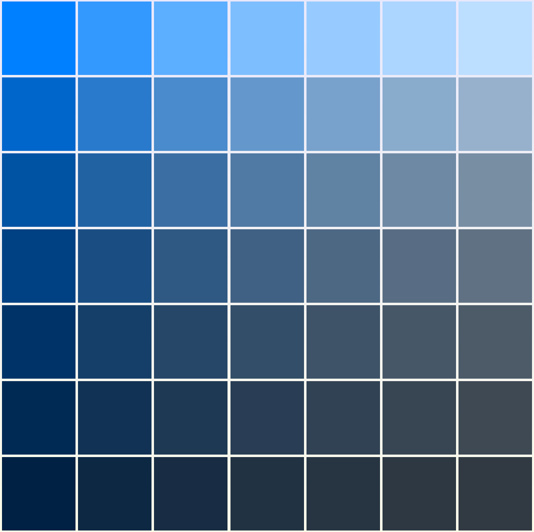
Hue: 180°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



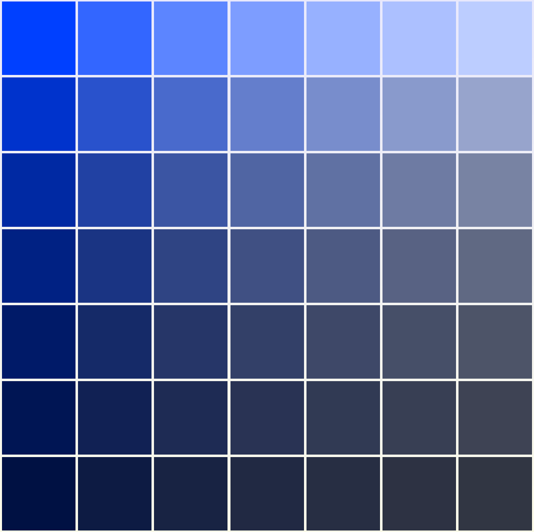
Hue: 195°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



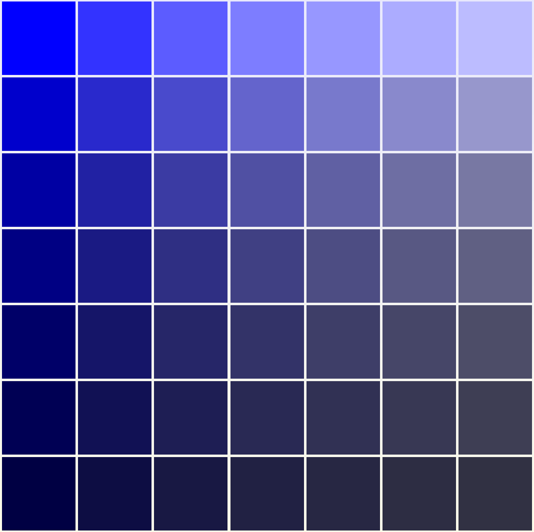
Hue: 210°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



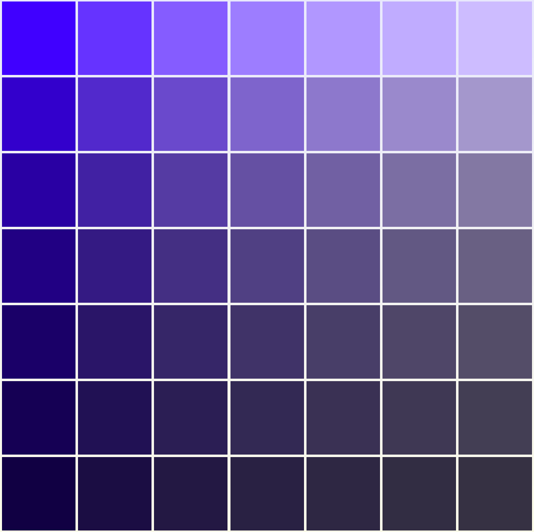
Hue: 225°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



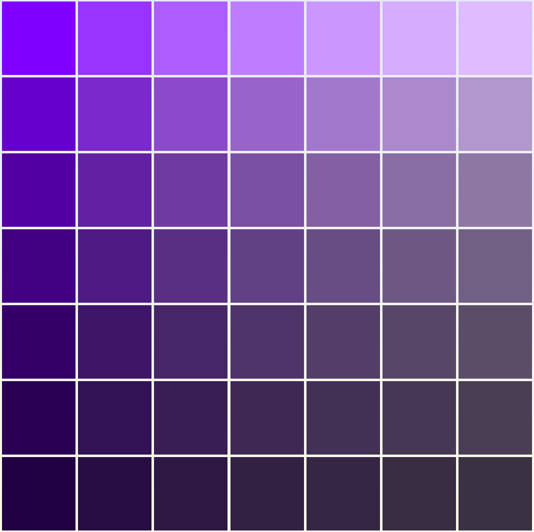
Hue: 240°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



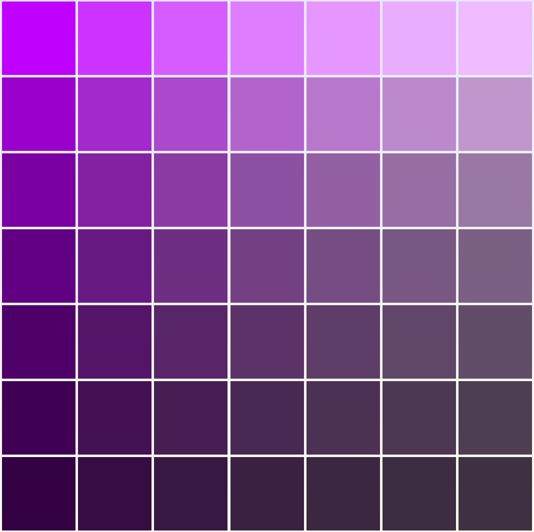
Hue: 255°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



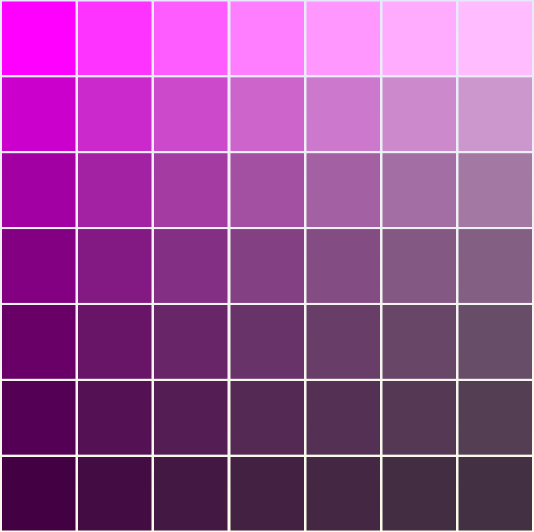
Hue: 270°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



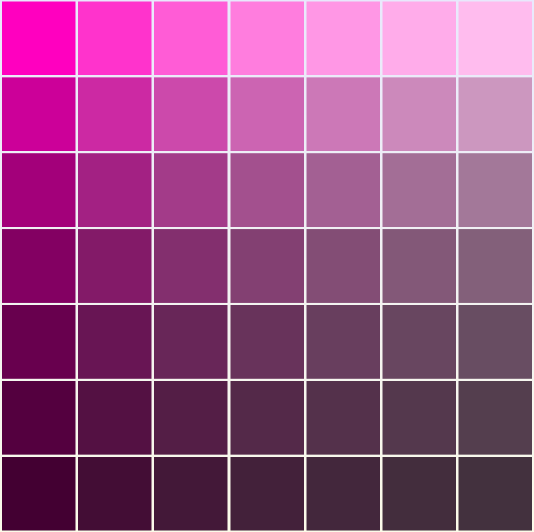
Hue: 285°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



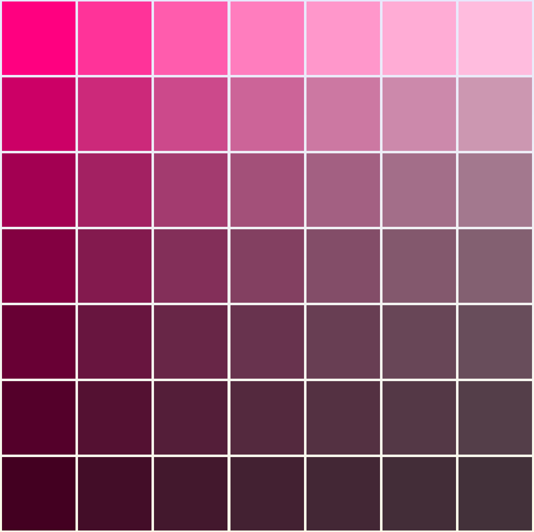
Hue: 300°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



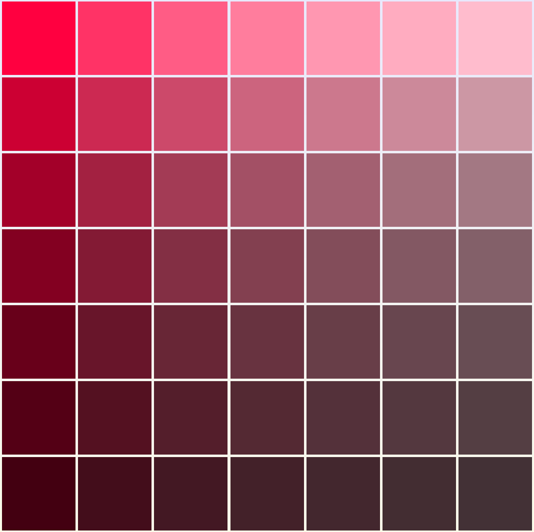
Hue: 315°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



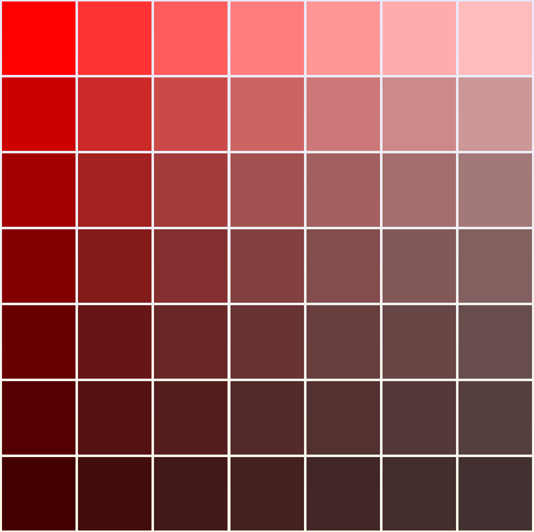
Hue: 330°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



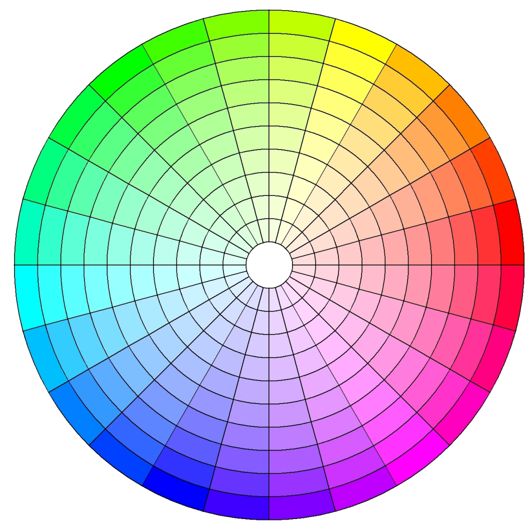
Hue: 345°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slab



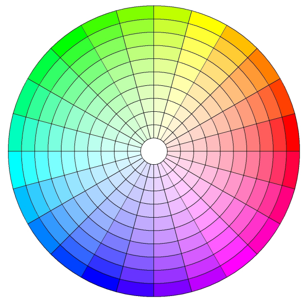
Hue: 360°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%

HSB Cylinder Slice



Ring saturation decreases by 20% toward the center.
Brightness is 1.

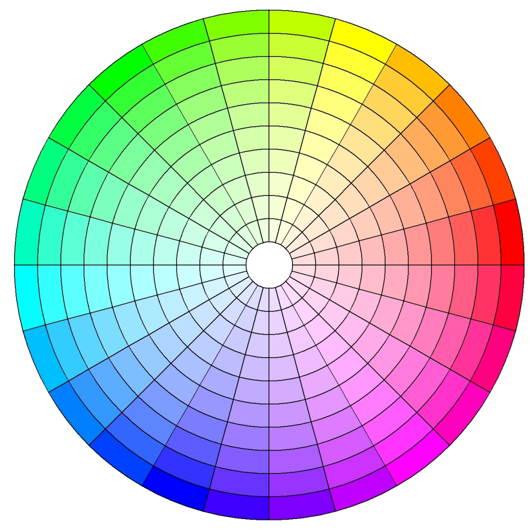
HSB Cylinder Slice



Ring Saturation decreases by 20% toward the center.
Brightness is 1.

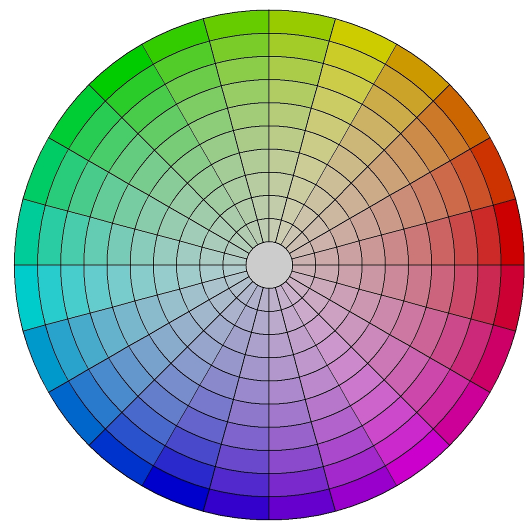
- Colors with the same brightness value are not perceptually the same brightness
- Fully saturated blue is perceived to be darker than fully saturated red or green
- Reds are more flashy than the blues

HSB Cylinder Slice



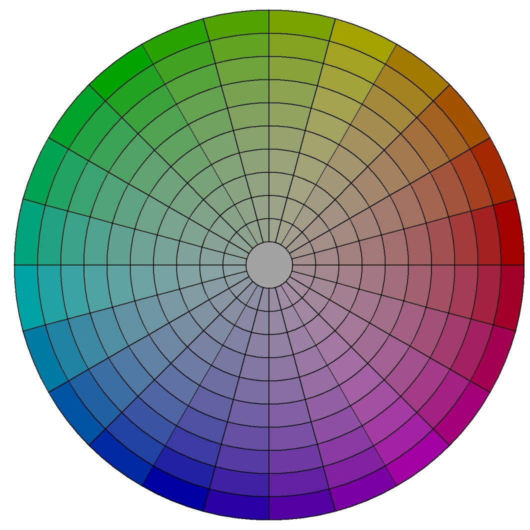
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^0 = 1$.

HSB Cylinder Slice



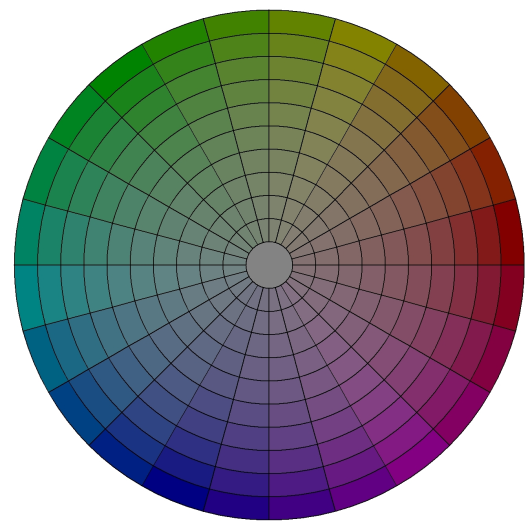
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^1 = .8$.

HSB Cylinder Slice



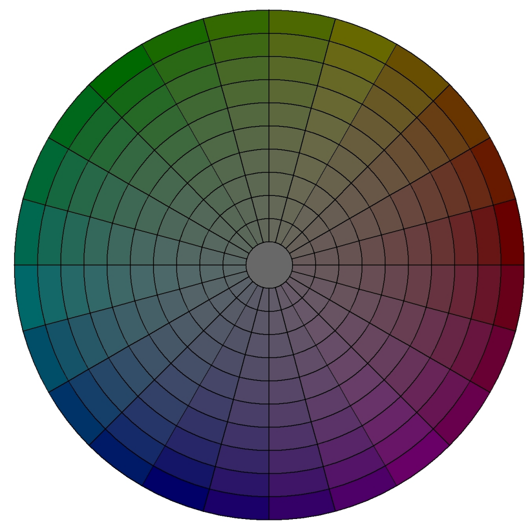
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^2 = .64$.

HSB Cylinder Slice



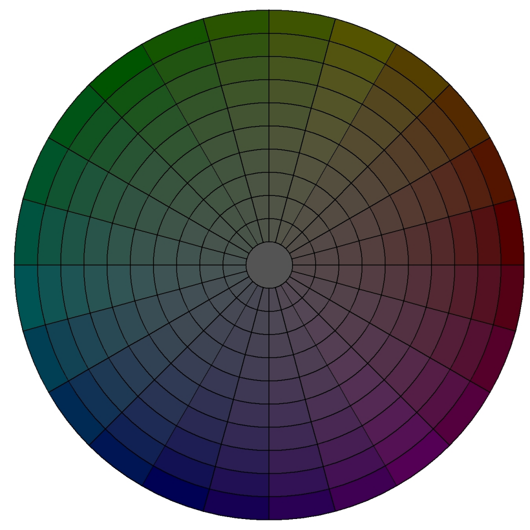
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^3 = .512$.

HSB Cylinder Slice



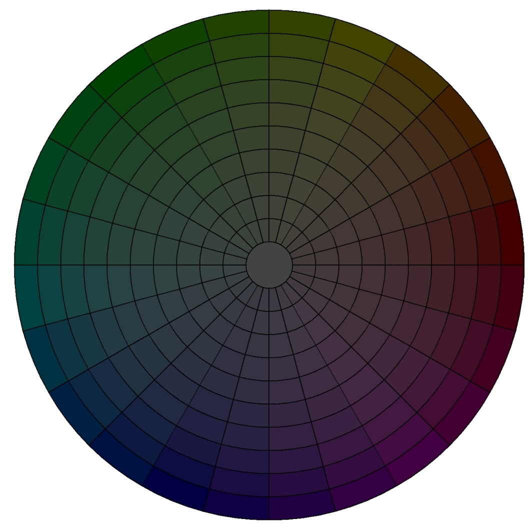
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^4 = .4096$.

HSB Cylinder Slice



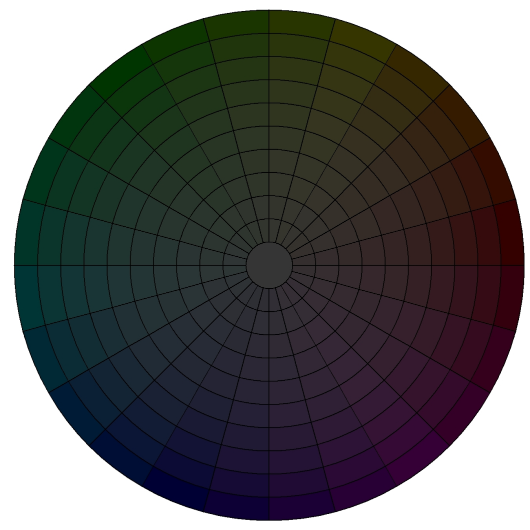
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^5 = .32768$.

HSB Cylinder Slice



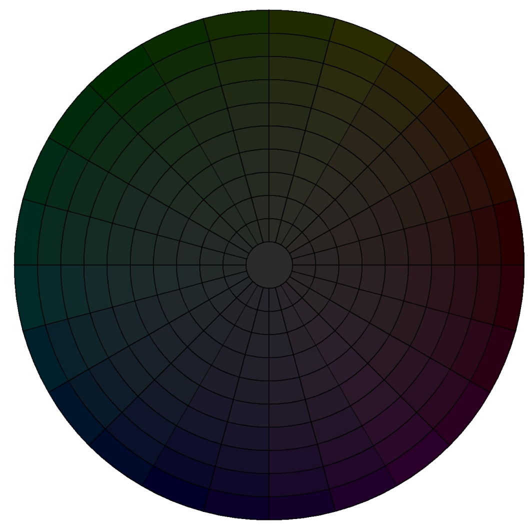
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^6 = .262144$.

HSB Cylinder Slice



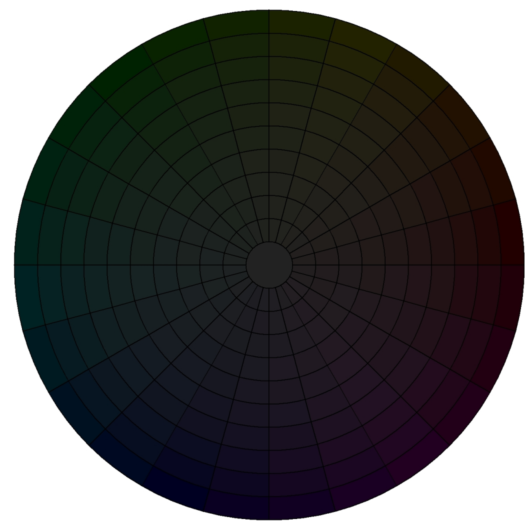
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^7 = .2097152$.

HSB Cylinder Slice



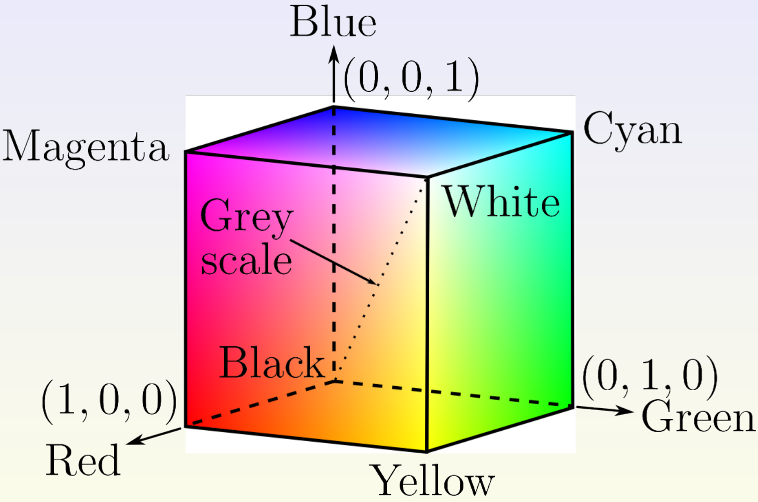
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $.8^8 = .16777216$.

HSB Cylinder Slice

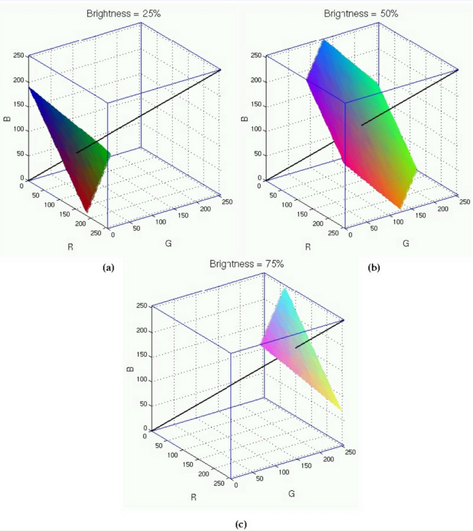


Ring saturation decreases horizontally by 20% in each ring toward the center. Brightness is $.8^9 = .134217728$.

RGB Color Cube

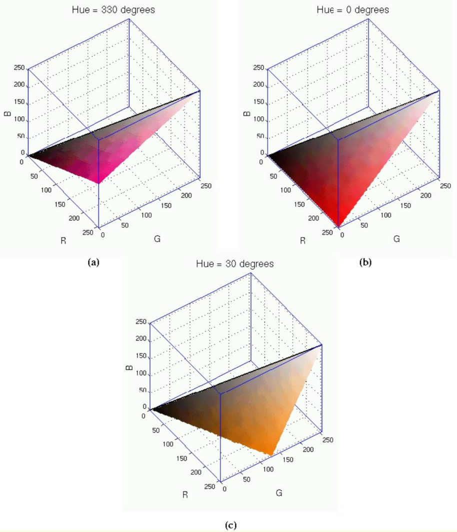


Planes Of Constant Brightness



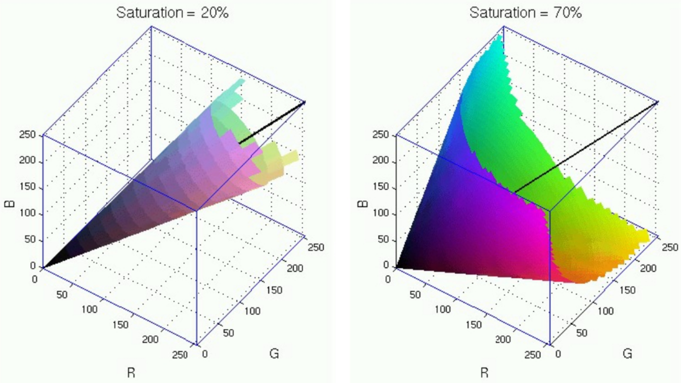
Brightness is here defined as R+G+B=constant.

Wedges Of Constant Hue



(a) (b) (c)

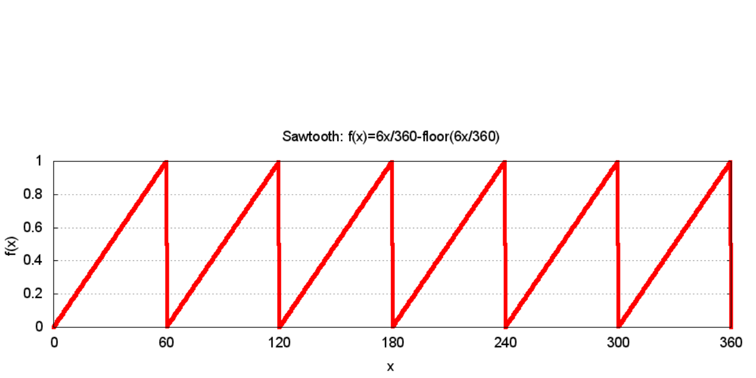
Cones Of Constant Saturation



The Sawtooth

For $0 \leq x \leq 360$

$$f(x) = x/60 - \lfloor x/60 \rfloor$$



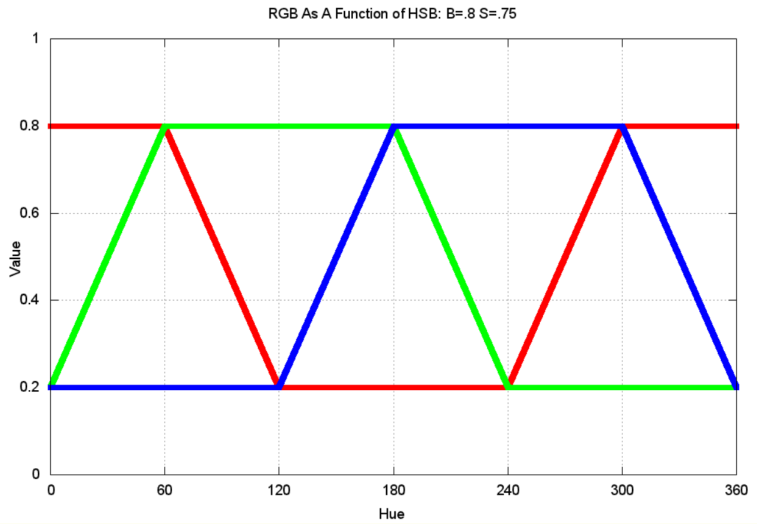
HSB to RGB

$$
\begin{aligned}
i(h) &= \left\lfloor \frac{h}{60} \right\rfloor \\
f(h) &= \frac{h}{60} - i(h) \\
c_{max} &= b \\
c_{min} &= c_{max}(1 - s) \\
q(h) &= c_{max}(1 - f(h)s) \\
t(h) &= c_{max}(1 - (1 - f(h))s) \\
(R, G, B) &= \begin{cases}
(c_{max}, t(h), c_{min}), & i(h) = 0 \\
(q(h), c_{max}, c_{min}), & i(h) = 1 \\
(c_{min}, c_{max}, t(h)), & i(h) = 2 \\
(c_{min}, q(h), c_{max}), & i(h) = 3 \\
(t(h), c_{min}, c_{max}), & i(h) = 4 \\
(c_{max}, c_{min}, q(h)), & i(h) = 5
\end{cases}
\end{aligned}
$$

HSB to RGB Code Sketch

```
hsb_to_rgb((float)h,(float)s,(float)b)
    i=(int)(h/60);
    f=h/60-i;
    mx=b;
    mn=mx*(1-s);
    q=mx*(1-f*s);
    t=mx*(1-(1-f)*s);
    switch(i)
      case 0:
           red=mx;
           green=t;
           blue=mn;
           break;
      case 1:
           red=q;
           green=mx;
           blue=mn;
           break;
      case 2:
           red=mn;
           green=mx;
           blue=t;
           break;
      case 3:
           red=mn;
           green=q;
           blue=mx;
           break;
      case 4:
           red=t;
           green=mn;
           blue=mx;
           break;
      case 5:
           red=mx;
           green=mn;
           blue=q;
           break;
    return(red,green,blue);
```

HSB to RGB


RGB As A Function of HSB: B=.8 S=.75
Value
Hue
0
0.2
0.4
0.6
0.8
1
0
60
120
180
240
300
360

Is There a Hue For Every Combination RGB

Let $(R, G, B) \in [0,1]^3$ be given. Let $c_{max} = \max\{R, G, B\}$ and $c_{min} = \min\{R, G, B\}$ There are six possibilities:

$$\begin{pmatrix} c_{max} \\ c_{min} \end{pmatrix} \in \left\{ \begin{pmatrix} R \\ G \end{pmatrix}, \begin{pmatrix} R \\ B \end{pmatrix}, \begin{pmatrix} G \\ R \end{pmatrix}, \begin{pmatrix} G \\ B \end{pmatrix}, \begin{pmatrix} B \\ R \end{pmatrix}, \begin{pmatrix} B \\ G \end{pmatrix} \right\}$$

The argument is similar for each of the cases. Consider the case

$$\begin{pmatrix} c_{max} \\ c_{min} \end{pmatrix} = \begin{pmatrix} R \\ G \end{pmatrix}$$

Then the hue angle θ satisfies $300^\circ \leq \theta \leq 360^\circ$. Whatever the value of B,

$$360 - \frac{B - c_{min}}{c_{max} - c_{min}} 60^\circ = \theta$$

General Case: f

$\max\{R, G, B\}$	$min\{R, G, B\}$	f	$h = atan2(\sqrt{3}(G - B), 2R - G - B)$
R	B	$\frac{G-B}{R-B}$	$60^\circ \left(\frac{G-B}{R-B}\right)$
R	G	$\frac{B-G}{R-G}$	$60^\circ \left(6 - \frac{B-G}{R-G}\right)$
G	R	$\frac{B-R}{G-R}$	$60^\circ \left(2 + \frac{B-R}{G-R}\right)$
G	B	$\frac{R-B}{G-B}$	$60^\circ \left(2 - \frac{R-B}{G-B}\right)$
B	R	$\frac{G-R}{B-R}$	$60^\circ \left(4 - \frac{G-R}{B-R}\right)$
B	G	$\frac{R-G}{B-G}$	$60^\circ \left(4 + \frac{R-G}{B-G}\right)$

HSB: Hue Equivalence Classes

$$R_0 = \max\{R_0, G_0, B_0\} \text{ and } B_0 = \min\{R_0, G_0, B_0\}$$

- Brightness: b
- Saturation: s
- Green Fraction: $f = \dfrac{G_0 - B_0}{R_0 - B_0}$
- Equivalence Class: $\begin{bmatrix} R_0 \\ G_0 \\ B_0 \end{bmatrix}$

$$\begin{bmatrix} R_0 \\ G_0 \\ B_0 \end{bmatrix} = \left\{ \begin{pmatrix} R \\ G \\ B \end{pmatrix} \in [0,1]^3 \;\middle|\; \text{for some } b, s \in [0,1], \begin{pmatrix} R \\ G \\ B \end{pmatrix} = b(1-s)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + bs\begin{pmatrix} 1 \\ f \\ 0 \end{pmatrix} \right\}$$

Hue Equivalence Class

Note that when

$$\begin{aligned}
f &= \frac{G_0 - B_0}{R_0 - B_0} \\
b &= R_0 \\
s &= \frac{R_0 - B_0}{R_0} \\
b(1-s) &= B_0 \\
bs &= R_0 - B_0 \\
\begin{pmatrix} R_0 \\ G_0 \\ B_0 \end{pmatrix} &= b(1-s)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + bs\begin{pmatrix} 1 \\ f \\ 0 \end{pmatrix}
\end{aligned}$$

Hue Equivalence Class

Proposition

For $0 \leq \lambda \leq 1$,

$$\lambda \begin{pmatrix} R_0 \\ G_0 \\ B_0 \end{pmatrix} \in \begin{bmatrix} R_0 \\ G_0 \\ B_0 \end{bmatrix}$$

Proof.

$$\lambda \begin{pmatrix} R_0 \\ G_0 \\ B_0 \end{pmatrix} = (\lambda b)(1-s) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + (\lambda b)s \begin{pmatrix} 1 \\ f \\ 0 \end{pmatrix}$$

□

HSB

- HSB is not perceptually uniform when saturation and brightness are held constant
  - With brightness and saturation held constant, the perceived difference between colors is not proportional to their separation angle in hue

The saturation and brightness are set to 1. The difference in hue is 60° between each pair. One pair is perceptually more similar than the other pair.

HSB

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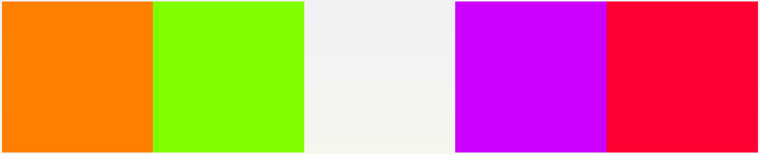


30.0 90.0 288.0 348.0

The saturation and brightness are set to 1. The difference in hue is 60° between each pair. One pair is perceptually more similar than the other pair.

Ewald Hering

- German physiologist and psychologist
- A founder of modern visual science
- Lived 1834-1918
- Taught at
  - Josephs-Akademie, Vienna (1865-1870)
  - University of Prague (1870-1895)
  - University of Leipzig (1895)
- Ideas based on color experience
- Postulated three types of color axes
  - Yellow-Blue
  - Red-Green
  - Black-White
- Opponent Color Processing
- Apparently a contradiction to Trichromatic Theory of Young and Helmholz

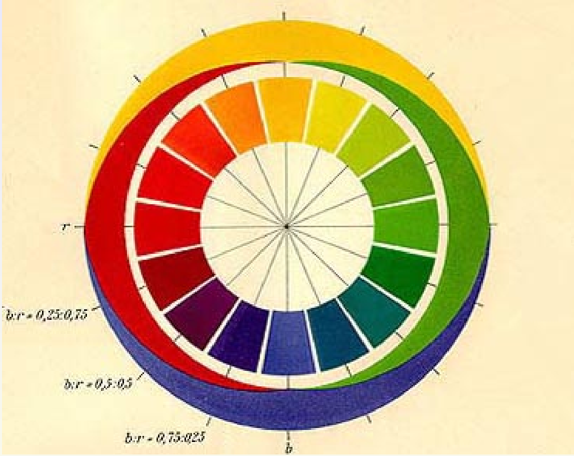
Hering's Evidence for Opponent Color Processing

- Yellow is psychologically just as basic as red, green, or blue
- Dichromats who lack red or green cones cannot see red or green colors but can see yellow
- Most colors of the spectrum seem to shift in hue as they brighten or darken
- These shifts do not happen in blue, green, or yellow colors

Hering's Opponent Processes

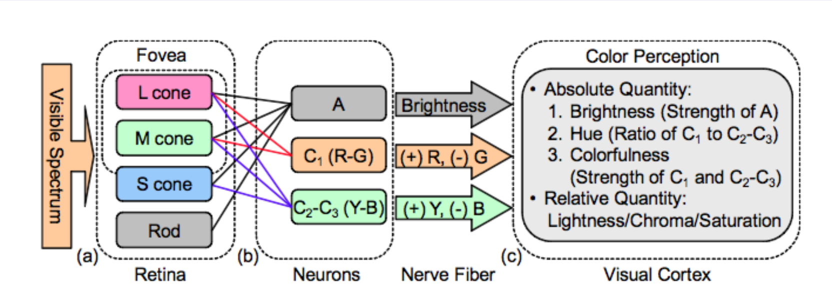
- red mixed with yellow yields orange
- red mixed with blue yields purple
- red mixed with green never creates the sensation of reddish green or greenish red
- blue mixed with yellow never creates the sensation of bluish yellow or yellowish blue

Hue Circle Explained by Hering's Opponent Processes



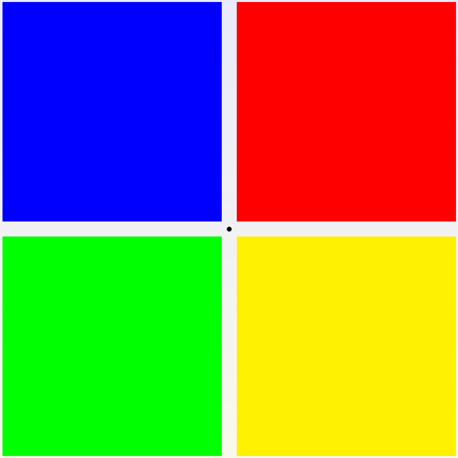
25% blue with 75% red produces crimson red
75% blue with 25% red produces blue violet
50% blue with 50% red produces violet

Opponent Model



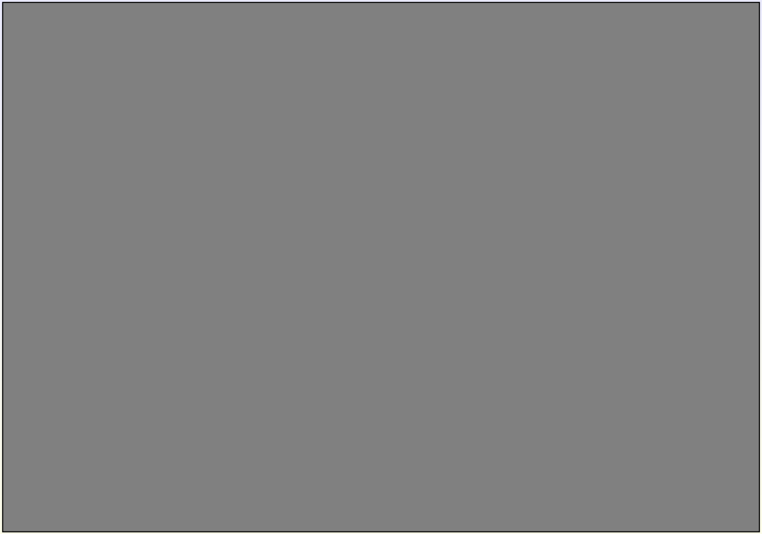
The chromatic processing behind the retina.

Before Image

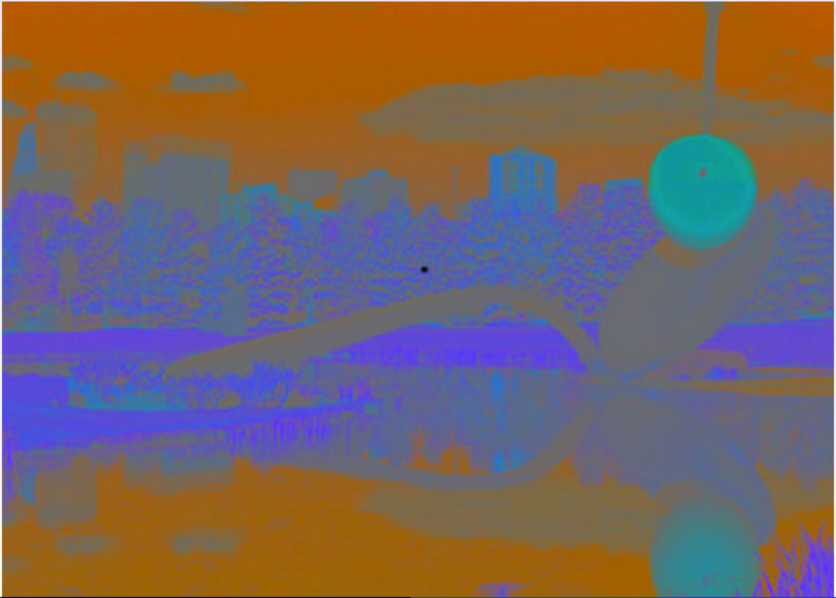


Keep eyes concentrated on black dot for 20 seconds.

After Image



Before Image



After Image

Color Blindness

- Typically genetically inherited throught X-chromosome
  - Genes that produce the photopigments are on X-chromosome
- Missing or Faulty cones
  - L-cones: (First) Protanopia, Protanomaly
  - M-cones: (Second) Deuteranopia, Deuteranopaly
  - S-cones: (Third) Trianopia, Trianopaly
- Deficiencies in M-cones or L-cones is commonly called red-green color blindness
- Deficiencies in S-cones is commonly called blue-yellow color blindness

Color Blindness

Twelve examples of pairs of horizontal blocks between which color blind people may not be able to distinguish.

Color Blindness Prevalence

Male	8.04%
Female	.44%

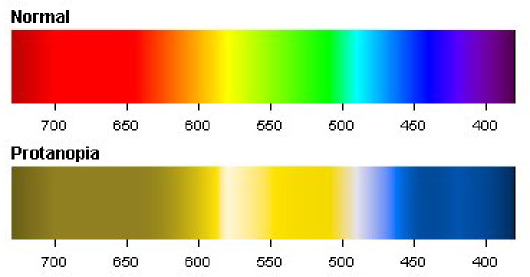
Gender	Protanopia	Protanomaly
Male	1.00%	1.3%
Female	0.02%	0.02%

Gender	Deuteranopia	Deuteranomaly
Male	1.0%	5.0%
Female	0.01%	0.35%

Gender	Tritanopia	Tritanomaly
Male	.001%	.01%
Female	.03%	0.01%

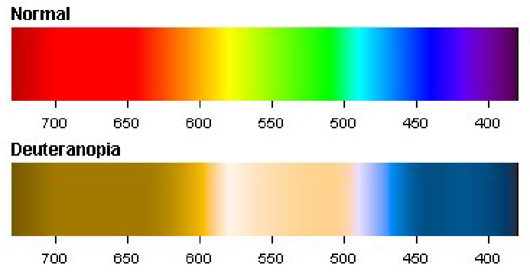
source: http://www.color-blindness.com/general/prevalence

Color Blindness: Protanopia



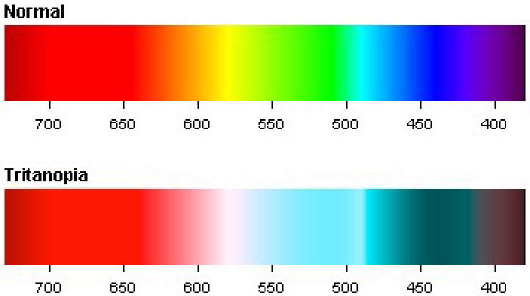
Protanopia: Deficient L-cones

Color Blindness: Deuteranopa



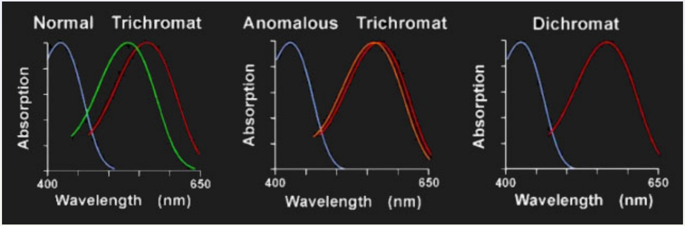
Deuteranopa: Deficient M-cones

Color Blindness: Trianopia



Trianopia: Deficient S-cones

Spectral Sensitivity Curves



Experience Color Blindness

(a) Normal

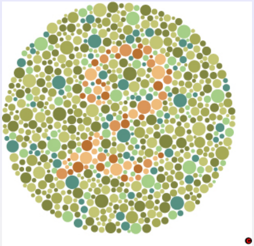
(b) Protoanopia

(c) Deuteranopia

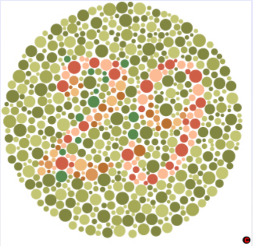
(d) Tritanopia

See http://www.vizcheck.com to make the simulated appearances

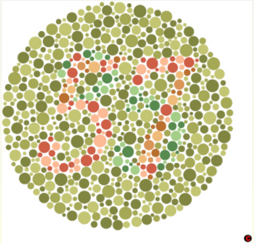
Ishihara Color Blind Test



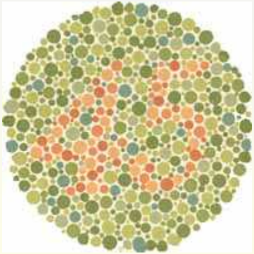
(e) ?



(f) spots

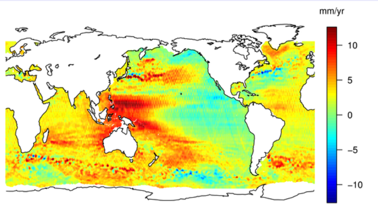


(g) 35

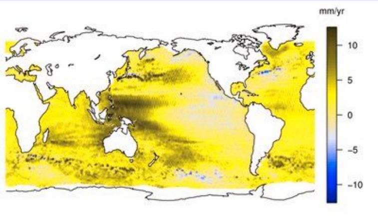


(h) spots

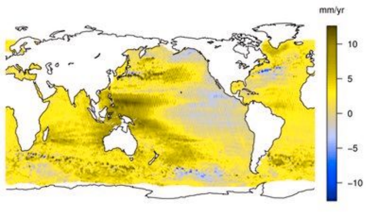
End Of the Rainbow Scale



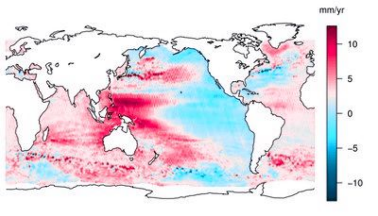
(i) Rainbow



(j) Protoanopia



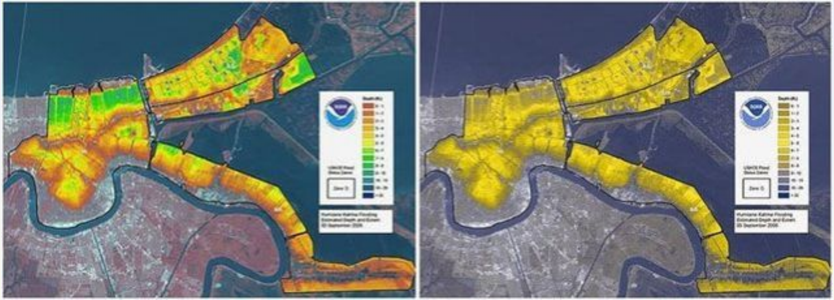
(k) Deuteranopia



(l) Tritanopia

See http://www.vizcheck.com to make the simulated appearances

End Of the Rainbow Scale



Normal on left; Red-green color blindness on right